

## Chapter 7 Factors Significantly Affecting Dynamic Response

### 7-1. Evaluation Procedure and Objectives

There are many important factors in a dynamic stress analysis that can greatly affect the response of a dam. The influence which the various material and strength parameters and loads have on the final results must be evaluated. This can be done by executing the model using a typical dam cross-section and typical material properties, then modifying the loads and parameters one-by-one to give an indication of the influence each factor has on the dynamic response. Once the important factors have been identified, the design effort should concentrate on the more critical factors that form the input to the dynamic analysis. Following is a discussion of the impact some of the parameters have on the response of a dam.

### 7-2. Design Response Spectra

*a. Spectral shape.* Both the shape of the spectrum and the PGA used to anchor the spectrum affect the dam response and should be established carefully. The dynamic response in a linear-elastic analysis is directly proportional to the PGA, but minor changes in the shape of the spectra may not result in proportional changes in the response.

*b. Comparison of standard spectra.* For comparison purposes, three widely accepted standard design response spectra will be considered, each representing the same site conditions. The design spectra are: (1) Applied Technology Council spectrum for rock of any characteristic whether shale-like or crystalline in nature (ATC 1984), (2) H. B. Seed spectrum for rock based on 28 records (Seed 1974), and (3) Newmark-Hall spectrum using recommended values for maximum ground velocity and displacement for competent crystalline rock (Newmark and Hall 1987). Figure 7-1 shows all three spectra normalized to 1.0 g PGA for the same rock foundation site conditions. The Newmark-Hall spectrum is based on the median or 50th percentile cumulative probability, where the other two spectra are based on the mean of the records used in their development. This difference in probability level is reflected in the spectral shape. The primary cause for the difference in

shape of these three spectra can be attributed to the assumptions and techniques used in smoothing the jagged spectra produced from the statistical combination of real earthquake records.

*c. Spectral accelerations.* Referring to Figure 7-1, the range of interest of natural period would be for periods of less than 1.0 second. This range would cover the mode shapes that produce significant response. In this range the spectral acceleration values for a given period vary between spectra up to as much as 65 percent. The ATC spectrum envelopes the other two design spectra, and is recommended for use as the standard design response spectrum. In linear-elastic response spectrum analyses, dynamic response of a particular system evaluated by two different response spectra is directly proportional to the spectral ordinates taken from the two spectra at the natural period of the system. Thus the shape of the design response spectrum greatly influences the results of the dynamic analysis.

### 7-3. Dam-Foundation Interaction, Damping Effect

*a. Properties of the foundation.* The two properties of the foundation rock that have a significant influence on the dynamic response are the damping ratio and the deformation modulus. The damping characteristics of the foundation contribute significantly to the damping of the combined dam-foundation system and must be considered in the analysis. When the foundation deformation modulus is low, the damping ratio of the combined system is considerably higher than the damping ratio of the RCC dam structure alone.

*b. Effective damping ratio.* There are two sources of damping for the foundation rock: (1) material (hysteretic) and (2) radiation. In contrast to this type of damping is the viscous type of damping (directly proportional to velocity) used in producing design response spectra. Therefore, it is necessary to develop an effective viscous damping ratio to represent the combined dam-foundation system in a response spectrum analysis. This is accomplished by using the curves provided in Figure D-6 of Appendix D, and the following equation is for an empty reservoir condition which allows the effects of foundation damping to be isolated. This method, developed by A. K. Chopra, is based on the

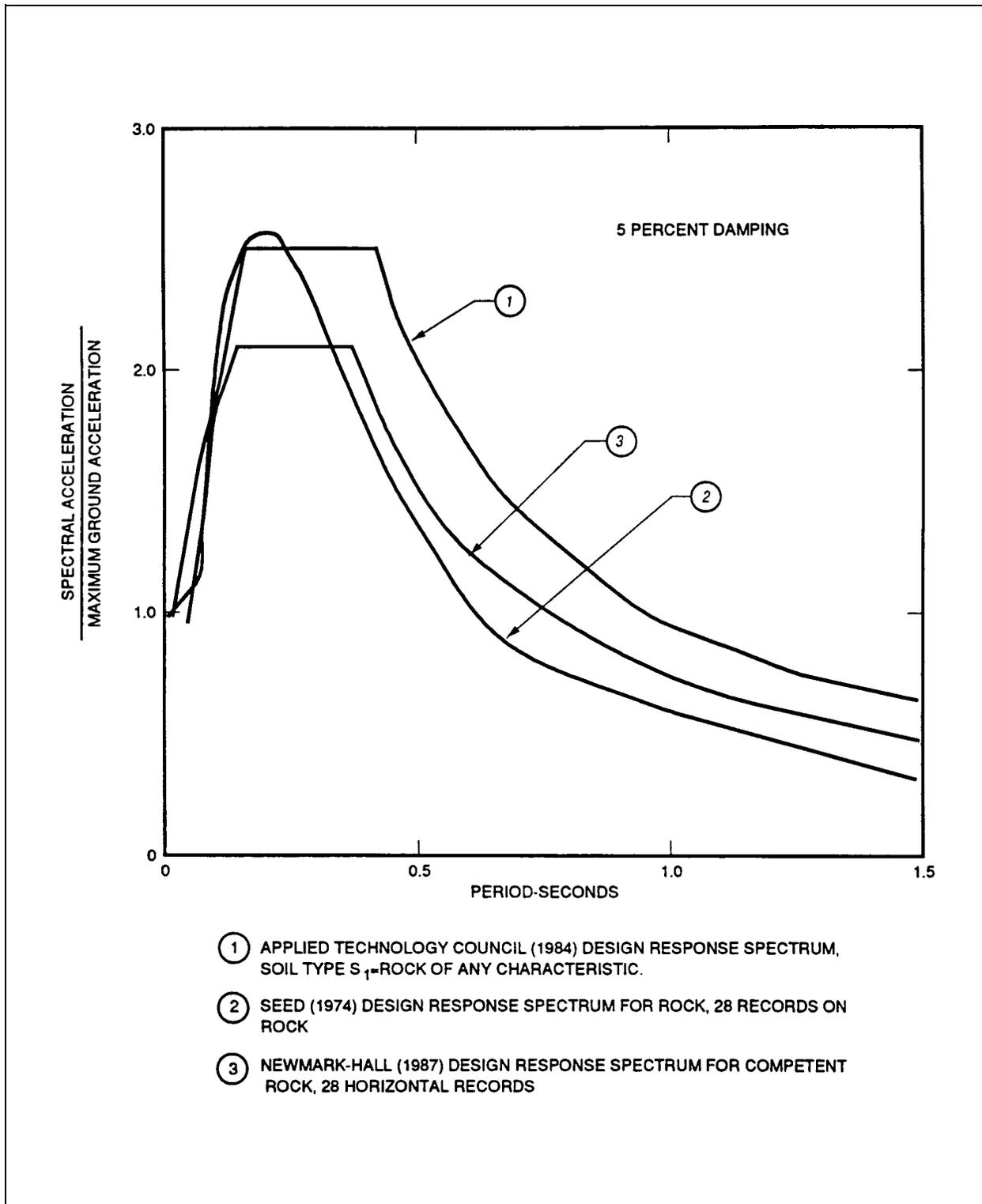


Figure 7-1. Comparison of design response spectra for rock foundations

fundamental mode of vibration, and has been shown to be reasonably close for the significant higher vibration modes (Fenves and Chopra 1986). In Figure D-6, damping for the foundation rock is expressed by the constant hysteretic damping factor.

$$\bar{\xi}_1 = \frac{1}{(R_f)^3} \xi_1 + \xi_f$$

where

$\bar{\xi}_1$  = the effective viscous damping ratio for the empty reservoir condition

$\xi_1$  = the viscous damping ratio for the RCC dam structure only

$$\begin{aligned} \xi_1 &= 5.0 \text{ percent for the OBE} \\ \xi_1 &= 7.0 \text{ percent for the MCE} \end{aligned}$$

$R_f$  = ratio of the fundamental period of the dam on a rigid foundation to the fundamental period of the dam on a foundation with a deformation modulus =  $E_f$

$\xi_f$  = added damping ratio due to dam-foundation rock interaction taken from Figure D-6

*c. Effect of damping on response.* To determine the effect that the damping ratio has on the response of a dam, the fundamental frequency of the composite finite element dam-foundation model must be determined. It is noted that for the response spectrum method, the effects of damping are contained only in the response spectrum itself. Thus, the ratio of the response of a dam/foundation system responding at one damping factor to the same system responding at a second damping factor is equal to the ratio of the spectral ordinates taken from the two spectra evaluated at the fundamental frequency of the system.

*d. Conclusion.* The damping characteristics of the foundation can have a great influence on the dynamic response. This indicates the need to carefully determine the value of the constant hysteretic damping factor for the foundation rock. This can be determined from experimental tests of appropriate rock samples subject to harmonically varying stress and strain. From such tests, the inelastic energy lost and the strain energy stored per cycle are determined and the hysteretic damping factor is calculated.

#### 7-4. Dam-Foundation Interaction, Foundation Modulus Effect

*a. Modulus of deformation.* The flexibility of the jointed rock foundation is characterized by the modulus of deformation which represents the relationship between applied load and the resulting elastic plus inelastic deformation. It is best determined by in-situ testing, but may be estimated from the elastic modulus of the rock by applying an appropriate reduction factor. In a linear-elastic analysis, the modulus of deformation is synonymous with Young's modulus of elasticity ( $E_r$ ).

*b. Dynamic characteristics affected.* The elastic modulus of the foundation influences the response because it directly affects the following dynamic characteristics of the dam-foundation system:

(1) Modal frequencies. As the modulus of deformation decreases, the modal frequencies of the composite dam/foundation system also decrease.

(2) Mode shapes. As the modulus of deformation decreases, the mode shapes are affected by increased rigid body translations and rotation of the dam on the elastic foundation.

(3) Effective damping ratio. As the modulus of deformation decreases, the effective damping ratio of the dam/foundation system increases.

*c. Effect of foundation modulus on response.* To determine the effect of the foundation modulus on dynamic response, a typical dam model was analyzed on foundations that bracket a wide range of foundation stiffness from infinitely stiff ( $E_s/E_r = 0.0$ ), to relatively flexible ( $E_s/E_r = 2.5$ ). The response was expressed as the distributed lateral inertia loading acting over the full height of the dam. Figure 7-2 shows the response graphically for three different values of  $E_s/E_r$ . It is noted that the total inertia load, or base shear, only varied by 15 percent, but a considerable variation occurred in the load pattern. As the foundation becomes more flexible, the greatest inertia load shifts from the upper portion of the dam to the lower portion. This would be accompanied by a considerable change in the concrete stresses.

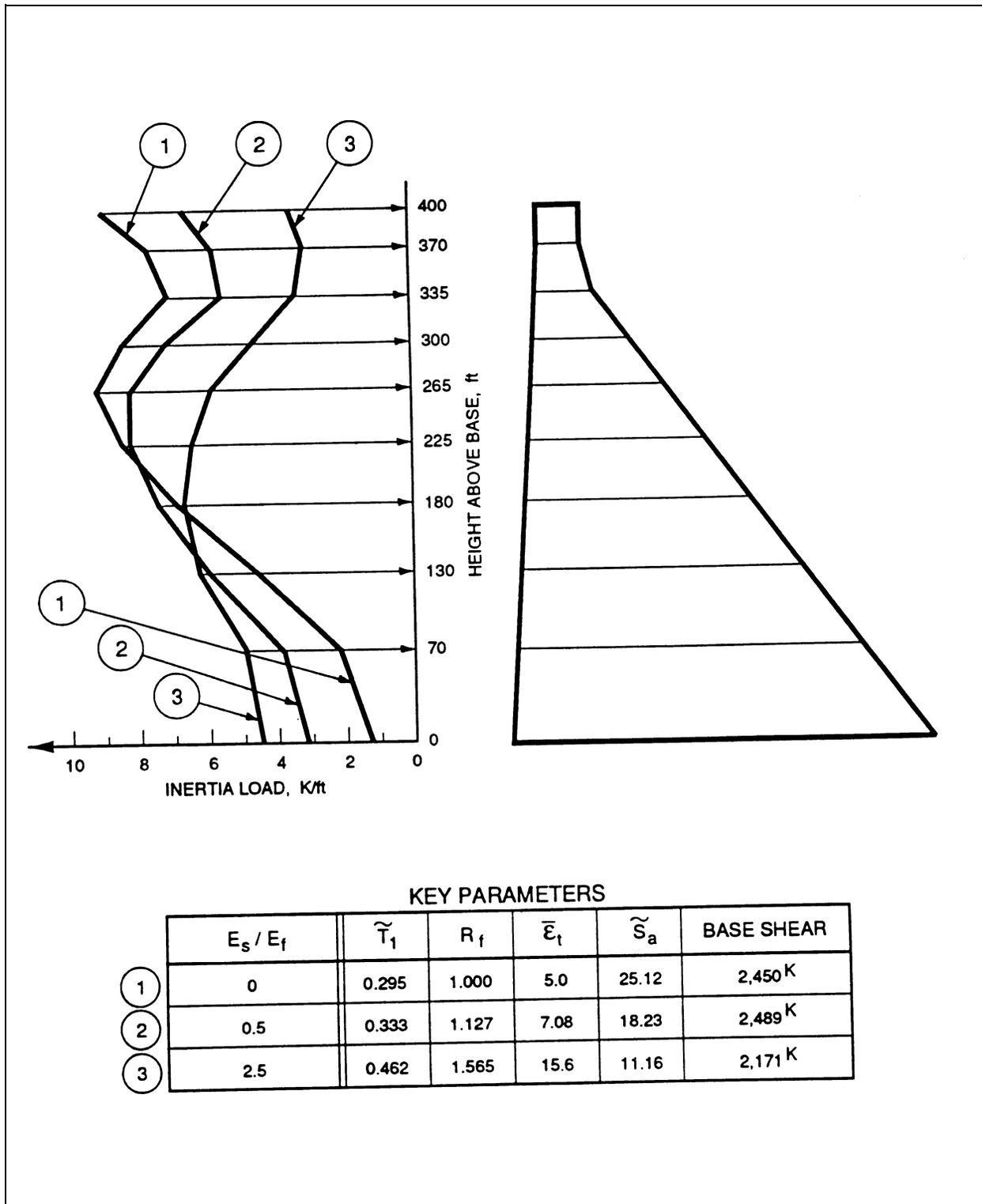


Figure 7-2. Fundamental mode response expressed as the distributed lateral inertia load for various foundation stiffnesses

## 7-5. Hydrodynamic Effect

*a. Dynamic characteristics affected.* Hydrodynamic load results from the interaction of the reservoir and the structural mass of the dam in response to ground motion. The dam-reservoir interaction changes the water pressure acting on the face of the dam, and directly affects the following dynamic characteristics of the system:

(1) Modal frequencies. As the depth of the reservoir increases beyond a depth equal to about one half of the height of the dam, there begins to be a noted decrease in the modal frequencies.

(2) Mode shapes. The equivalent added mass to account for reservoir effects, as discussed in paragraph 7-5c, changes the relative distribution of mass in the system. Thus, the normalized mode shapes will be affected to some degree.

(3) Effective damping ratio. As the depth of the reservoir increases, dam-reservoir interaction tends to increase the effective damping ratio.

*b. Added mass based on Westergaard's formula.* Accounting for hydrodynamic effects when using a composite finite element model (refer to paragraph 8-1d(3)(a)) requires developing an equivalent mass system which strategically adds mass to the dam-foundation model. The amount and location of the added lumped masses must be such that they correctly alter the dynamic properties described above in a manner which will also produce the desired pressure changes. Often the added mass is calculated based on Westergaard's pressure diagram divided by the acceleration due to gravity to convert it from a distributed load to a distributed mass.

*c. Added mass based on Chopra's method.* A. K. Chopra's Simplified Analysis Procedure (Chopra 1978) uses an equivalent mass system to consider compressibility of water and the dynamic properties of the dam and reservoir bottom. Chopra suggests that the key parameter that determines the significance of water compressibility is

$$\Omega_r = \frac{\omega_1^r}{\omega_1}$$

where

$\Omega_r$  = water compressibility significance parameter

$\omega_1^r$  = fundamental frequency of the impounded water idealized by a fluid domain of constant depth and infinite length

$\omega_1$  = fundamental frequency of the dam alone

and when

$\Omega_r \leq 0.5$ , compressibility of water is significant and should be accounted for in determining the hydrodynamic effect

*d. Standard pressure function curves.* In Chopra's system, the hydrodynamic pressure distribution and equivalent mass system are derived using a set of standard hydrodynamic pressure function curves. The equivalent mass system for the composite finite element method may be developed using the same principles as those for the Simplified Procedure. The added mass is determined by using the appropriate pressure function curve, certain equations from Chopra's Simplified Procedure, and the fundamental mode shape and frequency obtained from the finite element analysis of the dam-foundation model. Some additional requirements applying to added mass are discussed in paragraph 7-8c, and complete details for deriving the equivalent mass system for the composite finite element method are provided in Appendix D of this EP.

*e. Hydrodynamic pressure distribution.* Figure 7-3 shows the hydrodynamic pressure distribution associated with the fundamental mode for a typical dam with a high reservoir condition. Plot 1 shows the distribution calculated by Chopra's Simplified Procedure, where Plot 2 and Plot 3 were obtained using the composite finite element method with equivalent mass systems as discussed above. The added mass for Plot 2 was based on Westergaard's formula, and the added mass for Plot 3 was based on the standard pressure function curves and the method described in Appendix D. To extract the hydrodynamic pressure distribution using the composite finite element method, the dynamic analysis was

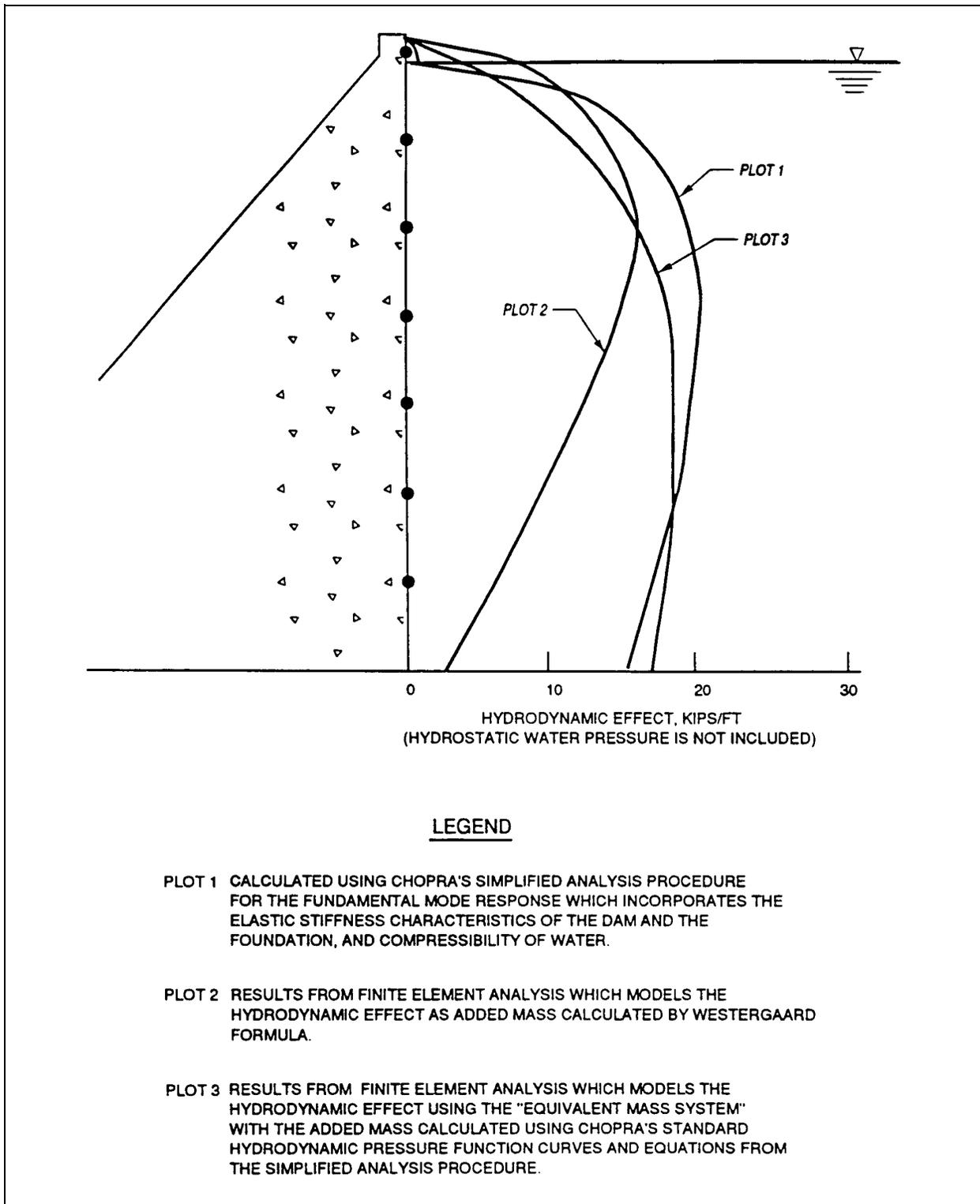


Figure 7-3. The hydrodynamic effect expressed as an "equivalent applied static pressure"

first performed on the dam-foundation model without added mass (which represents the empty reservoir condition), and then a second dynamic analysis was performed on the model with added mass. The difference in the unbalanced nodal forces between these two analyses represented the hydrodynamic forces exerted on the nodes. From these nodal forces the pressure distribution was readily determined.

*f. Comparison of hydrodynamic methods.*

Although Chopra's Simplified Procedure is only an approximate procedure based on the standard fundamental mode shape and simplified methods for determining the required periods of vibration, it is assumed that the procedure provides hydrodynamic loading that is at least within the general order of accuracy expected in dynamic analyses. On this basis, the equivalent mass system based on Westergaard's formula (Plot 2), underestimated the hydrodynamic loading on the typical dam section by about 40 percent. The equivalent mass system developed by the method described in Appendix D produced hydrodynamic loading (Plot 3) which correlated reasonably well with Chopra's Simplified Procedure. On this basis, the method described in Appendix D, which uses the standard pressure function curves, is recommended for developing the equivalent mass system.

*g. Hydrodynamic contribution to response.* For high pool conditions, a large portion of the dynamic response is attributable to the hydrodynamic effect. In the example that produced Plot 3 in Figure 7-3, 47 percent of the total equivalent mass system consisted of the added mass representing the hydrodynamic effects. Therefore, the equivalent mass system will significantly affect the response for pool depths greater than about half of the height of the dam.

## 7-6. Reservoir Bottom Absorption

*a. Wave reflection coefficient.* The nonrigid reservoir bottom partially absorbs incident hydrodynamic pressure waves. This moderates the increase in response of the dam due to the dam's interaction with the impounded water. This is readily apparent by comparing the standard hydrodynamic pressure function curves for two different reservoir bottom absorption conditions. Reservoir bottom absorption is expressed by a wave reflection coefficient which varies from zero for a fully absorptive condition to 1.0 for a fully reflective condition. Figure D-4 in

Appendix D shows the pressure function curves for reservoir bottom conditions with wave reflection coefficients of 0.50 and 0.75. As apparent from these curves, the hydrodynamic pressure increases with an increase in the reflection coefficient.

*b. Effects of  $R_w$ .* When the fundamental vibration period of impounded water and the fundamental period of the dam are approximately equal,  $R_w$  approaches 1.0. This condition indicates the approach of a state of resonance, and the pressure function then becomes quite large for a nonabsorptive reservoir bottom. In contrast, the pressure function for an absorptive bottom is much less affected by the approach of resonance, because the effect of reservoir bottom absorption is to reduce the large resonant displacement peaks.

*c. Estimating reservoir bottom absorption.*

Assuming a nonabsorptive reservoir bottom may lead to an overly conservative hydrodynamic response for dams when the earthquake load condition includes a high forebay pool. The degree of adsorptiveness characterized by the wave reflection coefficient is usually difficult to determine reliably. The value of the wave reflection coefficient will likely increase during the life of the dam as sediments are continuously deposited. Therefore, it is recommended that the effects of reservoir bottom absorption be included in the dynamic analysis by using a wave reflection coefficient based on the properties of the impounded water and the foundation rock, and neglect the additional adsorptiveness due to sediments that will eventually be deposited (Fenves and Chopra 1984). The wave reflection coefficient is determined by the following equation:

$$\alpha = \frac{1 - k}{1 + k}$$

where

$\alpha$  = wave reflection coefficient

$k = \rho C / \rho_r C_r$

$\rho$  = mass density of water = 1.938 (lb-sec<sup>2</sup>) /ft<sup>4</sup>

$C$  = velocity of pressure waves in water = 4,720 ft/sec

$\rho_r$  = mass density of the foundation rock in (lb-sec<sup>2</sup>) /ft<sup>4</sup>

$C_r$  = velocity of pressure waves in the  
foundation rock =  $12\sqrt{E_f/\rho_r}$

$E_f$  = deformation modulus of the foundation rock  
in lb/in.<sup>2</sup>

## 7-7. Method of Combining Modes

*a. Maximum modal responses.* The maximum modal response in a response spectrum analysis is the maximum possible contribution that a particular mode can make to the dynamic response. However, all the modes do not arrive at their maximums at the same point of time during the period of ground motion. Thus, for a single ground motion record there is one point in time when the maximum dynamic response is reached, and this maximum response is made up of various fractional parts of the individual maximum modal responses. The "fractional parts" are unique for each ground motion record. If a response spectrum analysis is made for a single ground motion record, the maximum dynamic response can only be approximated because the exact makeup of the "fractional parts" of the maximum modal responses cannot be computed. A time-history analysis is required to determine the exact solution for a given ground motion record.

*b. Statistical combination methods.* A smooth design response spectrum may be considered as a convenient representation of many possible ground motion records that could make up the design earthquake. As discussed in paragraph 5-5a, design response spectra are often referred to as statistical representations of the ground motion records used in their development (such as mean, median, 84th percentile). On a similar basis, the maximum modal responses of a response spectrum analysis are combined by statistical methods to produce a reasonable dynamic response to the many possible ground motions that could make up the design earthquake.

*c. Coupling coefficients.* Tables 7-1 and 7-2 present four commonly used mode combination methods. The difference in the methods is in the calculation of the coupling coefficient between modes. The coupling coefficients may be simple discrete functions as is the case with the square root of the sum of the squares method (SRSS) which treats the modal

responses as random variables. The functions may be more complex involving modal frequencies or both modal frequencies and damping factors as is the case with the complete quadratic combination method (CQC) and the double sum method (DSM). The more complex methods give additional accounting in the coefficient calculation when the frequencies of the two modes under consideration are close. Two closely spaced modes are coupled, and when one of the modes is excited, it tends to excite the other mode. However, the modal frequencies associated with gravity dams are normally fairly well separated.

*d. Comparing methods.* The base shear was the response parameter used for comparing the four combination methods. By using several load cases, foundation conditions, and damping ratios, eight sets of maximum modal base shear values were made available to test the combination methods. The more complex methods, CQC and DSM, increased the coupling coefficients for closely spaced mode which produced greater combined responses than the SRSS method. The spacing of the modal frequencies for the TPM was such that no two modes qualified for "additional accounting," so the combined response for the TPM is the same as SRSS. The two most often used methods are SRSS and CQC.

*e. Conclusion.* The mode combination method does not greatly affect the order of accuracy of the dynamic analysis. The factors discussed previously have far greater influence on the dynamic response. The preliminary design of new dams, and the final design of dams not considered to be under critical seismic conditions, may use either the SRSS or the CQC method. Final design of dams under critical seismic conditions and evaluation of existing dams shall use the more refined CQC method.

## 7-8. Vertical Component of Ground Motion

*a. Factors that contribute to the response.* It is very difficult to make a general assessment of the influence of the vertical component of ground motion on the total dynamic response because of the number of factors involved. The vertical component of ground motion can be significant under certain conditions. The most important factors that affect the contribution of the vertical component to the response are:

**Table 7-1**  
**Combining Modal Responses: Square Root of the Sum of the Squares Method (SRSS) and Ten Percent Method (TPM)**

STATISTICAL METHODS consider the phasing of the modes by utilizing a "coupling coefficient" between the various modes as expressed by the basic equation:

$$R = \left[ \sum_{i=1}^N \sum_{j=1}^N R_i P_{ij} R_j \right]^{1/2}$$

where:  $N$  = number of modes to be considered  
 $R$  = total modal response  
 $R_i$  = maximum modal response in the  $i$ th mode  
 $R_j$  = maximum modal response in the  $j$ th mode  
 $P_{ij}$  = coupling coefficient between modes  $i$  and  $j$

There are several methods for determining the  $P_{ij}$  values. They are given below in the order of complexity:

Method 1: Square Root of the Sum of the Squares (SRSS)

$$P_{ij} = \begin{cases} 1.0 & \text{if } i = j \\ 0.0 & \text{if } i \neq j \end{cases}$$

The basic equation then reduces to

$$R = \sum_{i=1}^N [R_i^2]^{1/2}$$

Method 2: Ten Percent Method (TPM)

$$P_{ij} = \begin{cases} 1.0 & \text{if } \frac{\omega_j - \omega_i}{\omega_i} \leq 0.1 \\ 0.0 & \text{if } \frac{\omega_j - \omega_i}{\omega_j} > 0.1 \end{cases}$$

where

$\omega_i$  = the natural frequency for the  $i$ th mode

$\omega_j$  = the natural frequency for the  $j$ th mode

This method gives additional accounting for modes with nearly the same frequency. If none exist, TPM reduces to SRSS.

**Table 7-2**  
**Combining Modal Responses: Complete Quadratic Combination Method (CQC) and Double Sum Method (DSM)**

Method 3: Complete Quadratic Combination (CQC)

$$P_{ij} = \left[ \frac{8\sqrt{\varepsilon_i \varepsilon_j} (\varepsilon_i + r\varepsilon_j)r^{3/2}}{(1 - r^2)^2 + 4\varepsilon_i \varepsilon_j r (1 + r^2) + 4(\varepsilon_i^2 + \varepsilon_j^2)r^2} \right]$$

where

$\varepsilon_i$  = modal damping ratio for the *i*th mode

$\varepsilon_j$  = modal damping ratio for the *j*th mode

$$r = \frac{\omega_j}{\omega_i}$$

This method is based on both modal frequency and modal damping. However, for design of gravity dams, there is no procedure available to establish reasonable damping ratios for the higher modes. The effective viscous damping factor calculated according to the recommended procedure in this EP is used for all modes.

Method 4: Double Sum Method (DSM)

$$P_{ij} = \left[ 1 + \left\{ \frac{(\omega'_i - \omega'_j)^2}{(\varepsilon'_i \omega_i + \varepsilon'_j \omega_j)} \right\}^2 \right]^{-1}$$

where

$$\omega'_i = \omega_i (1 - \varepsilon_i^2)^{1/2}$$

$$\omega'_j = \omega_j (1 - \varepsilon_j^2)^{1/2}$$

$$\varepsilon'_i = \varepsilon_i + \frac{2}{t_d \omega_i}$$

$$\varepsilon'_j = \varepsilon_j + \frac{2}{t_d \omega_j}$$

$$t_d = 10 \text{ seconds (earthquake duration)}$$

This method is similar to CQC, but is slightly more conservative.

Note: Refer to Table 7-1 for the basic equation for obtaining the total modal response, and for definition of terms not provided on this table.

(1) The PGA associated with the vertical component. In some instances the vertical component PGA may be as great or greater than the horizontal component PGA. Refer to paragraph 5-6a.

(2) The shape of the vertical component design response spectrum. The frequency content of the vertical component of ground motion is usually higher than the frequency content of the horizontal component. This causes the vertical spectrum shape to be different than the horizontal spectrum shape. The vertical component will excite modes in the lower frequency range less than will the horizontal component.

(3) The depth of the reservoir. Vertical ground motion causes hydrodynamic pressure waves to be generated which exert a lateral load against the face of the dam (this hydrodynamic load is in addition to that discussed in paragraph 7-5). When considering stresses caused by the vertical component of ground motion, the stress induced by the hydrodynamic pressure waves can be larger than the stress caused by the inertia response associated with the mass of the dam. For a nonabsorptive reservoir bottom, the hydrodynamic load theoretically reaches infinity at the natural vibration frequencies of the reservoir. This is in contrast to stresses caused by the horizontal component of ground motion where the stress caused by the hydrodynamic load is small compared to the stress caused by the inertia response associated with the mass of the dam.

(4) Reservoir bottom absorption. Reservoir bottom absorption greatly reduces the added hydrodynamic load due to vertical ground motion and eliminates the unbounded peaks in the response, described above, at excitation frequencies equal to the natural vibration frequencies of the reservoir.

*b. Method of analysis.* Except for the hydrodynamic load contribution which is discussed later, determining the response due to the vertical component of ground motion follows the same general procedures and recommendations that apply in determining the horizontal component response. The vertical component design response spectrum, and the PGA associated with vertical excitation are used to define the design earthquake. It should be noted that for vertical direction excitation, the fundamental mode and some or all of the significant higher modes are often different than for horizontal excitation. The

participation factor and the mode coefficient for a particular mode and direction of excitation may be used to judge the order of importance of the modes, and which modes will make a significant contribution to the dynamic response.

*c. Equivalent added mass system.* The added mass associated with the equivalent mass system discussed in paragraph 7-5c should be active in the horizontal direction, and inactive in the vertical direction. Added mass representing backfill or silt deposits against vertical or near vertical surfaces of the dam should also be active horizontally and inactive vertically. If the backfill is placed on the sloping face of the dam, the magnitude of the added mass acting vertically should be determined as described in paragraph 6-3b.

*d. Hydrodynamic loading.* The vertical component of ground motion causes hydrodynamic pressure waves to be generated from the reservoir bottom into the impounded water above. These pressure waves act horizontally against the vertical or near vertical face of the dam. In the composite finite element method, the equivalent mass system discussed in paragraph 7-5 accounts for the hydrodynamic reservoir effects caused by the horizontal component of ground motion, but it does not account for the effect of the hydrodynamic pressure waves generated by the vertical component of ground motion. To account for the effect of the pressure waves, a finite element-substructure model configuration is required as discussed in Chapter 8.

*e. Combining component responses.* The individual vertical and horizontal component dynamic responses are not in phase. They are independent maximum component responses that do not occur at the same point in time during the period of ground motion activity. Each pair of horizontal and vertical ground motion records representing a single earthquake event would have a unique phase relationship. Since the response spectrum method encompasses many possible ground motion events which make up the design earthquake, the maximum vertical and horizontal component responses are combined by a statistical method to produce a total dynamic response with reasonable probability of occurrence. It is recommended that the phasing of the two maximum component responses be treated as two unrelated random occurrences, and they be combined by the square root of the sum of the squares method (SRSS).

*f. Conclusions.* Under certain critical seismic conditions, the response to the vertical component of ground motion may be significant when compared to the response to the horizontal component; however the phase relationship will greatly moderate the vertical component contribution to the total response. On this basis, the vertical component of ground motion

may be ignored in the preliminary design of new dams not subject to critical seismic conditions. The vertical component of ground motion shall be included for preliminary designs subject to critical seismic conditions, all final designs, and evaluation of existing dams.