APPENDIX A

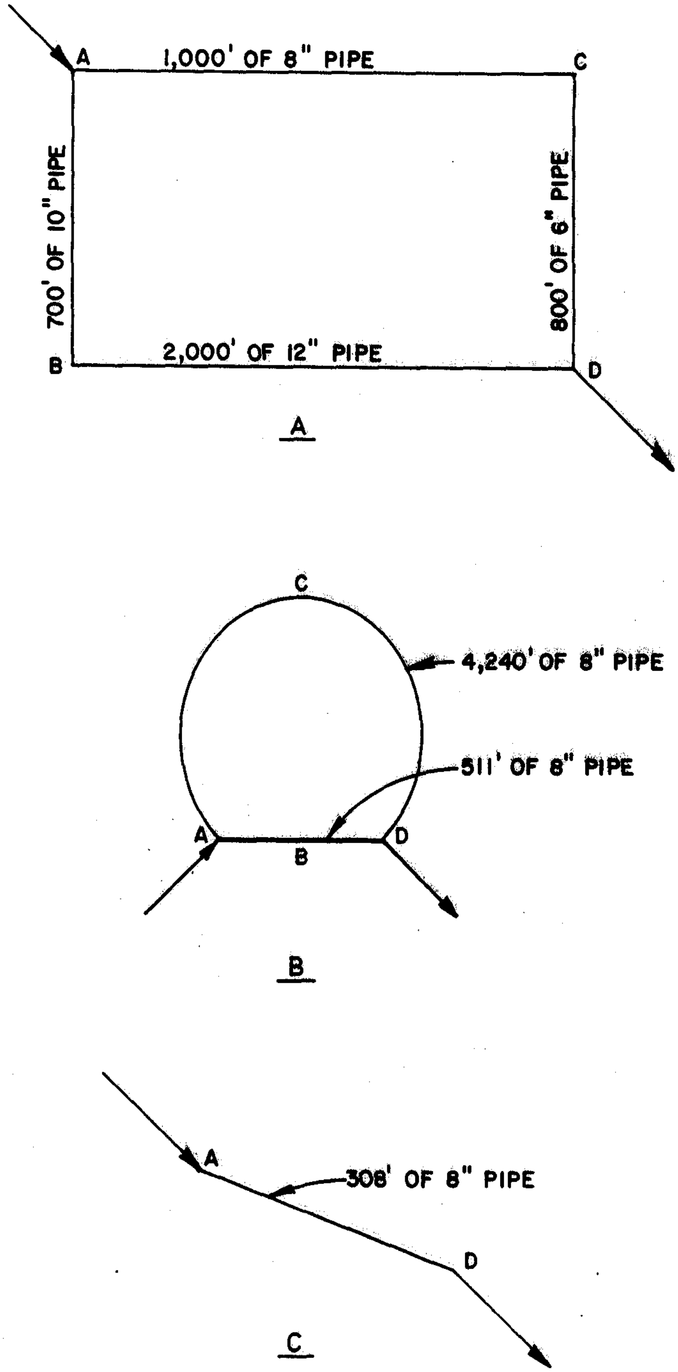
DISTRIBUTION SYSTEM HYDRAULIC ANALYSES

A-1. General. The sizing and location of mains, pump stations, and elevated storage facilites are dependent upon hydraulic analyses of the distribution system. The major techniques used in analysis of distribution networks are reduction into equivalent pipes and Hardy Cross analysis. For all but the very smallest systems, these analyses are best performed on computers.

A-2. Equivalent pipes. The "equivalent pipe" technique is a means of reducing a complex pipe network into a simpler configuration. It involves the substitution of one pipe of specific diameter and variable length or specific length and variable diameter for a series of different size pipes or several parallel pipes, as long as there are no inputs or withdrawals of water between the end points of the system. Application of the equivalent pipe method is best demonstrated by example. Referring to figure A-1, assume that the pipe network shown is to be converted to an equivalent 8-inch pipe. The following procedure should be used.

a. Series of different size pipes will be converted. An example is ACD and ABD in part (A) of figure A-1 being converted to equivalent 8-inch pipes. A flow rate will be assumed through each branch, the resulting loss of head calculated through the branch, and the length of 8-inch pipe substituted, which will give the same total loss of head through each branch. For example, assume that 200 gpm flows through branch ACD and 400 gpm through ABD. Using tables or nomographs based on the Hazen-Williams formula or the formula itself as given in equation 3-1, the loss of head through Section AC is 1.51 feet per 1,000 feet of pipe length (for this example, assume C = 100 for all pipes), so the total loss of head through pipe AC is (1.51/1,000) x 1,000 = 1.51 feet. Likewise, the loss of head through pipe CD at a flow of 200 gpm is 6.1 feet per 1,000 feet of pipe length, which gives a loss of head through CD of (6.1/1000) x 800 = 4.9 feet. Hence, the total loss of head through ACD is 6.4 feet. The length of 8-inch pipe which will have the same total loss of head at the same flow is 6.4 / (1.51/1000) = 4,240 feet. The two pipes of branch ACD can be replaced by 4,240 feet of 8-inch pipe. The total loss of head through ABD at a flow of 400 gpm is (1.83 x 700/1000) + (0.75 x 2,000/1,000) = 2.78 feet. At the same flow of 400 gpm, an 8-inch pipe has a loss of head of 5.44 feet per 1,000 feet of length, so the length of 8-inch pipe equivalent to section ABD is (2.78/5.44) x 1,000 = 511 feet. Part (B) of figure A-1 shows the configuration of the system after branches ACD and ABD have been converted to equivalent 8-inch pipes.

b. The 8-inch equivalent pipes for ACD and ABD will be converted into a single equivalent 8-inch pipe. Since it is known that water passing through ACD must have the same loss of head as water passing



U. S. Army Corps of Engineers

FIGURE A-1. EQUIVALENT PIPE NETWORKS

through ABD, a constant loss of head value can be assumed. For purposes of this example, a loss of head of 10 feet between A and D is arbitrarily chosen. At this total loss of head, the loss of head per 1,000 feet of length in ACD is 2.36 feet and in ABD is 19.6 feet. Referring again to the Hazen-Williams equation, it can be determined that the flows producing these losses of head are 255 gpm in ACD and 800 gpm in ABD. Thus, the total flow from A to D with a loss of head of 10 feet is 1,055 gpm. At this total flow, the loss of head through a single 8-inch pipe is 32.5 feet per 1,000 feet of length. For a total loss of head of 10 feet from A to D at a total flow of 1,055 gpm, a single 8-inch pipe would be (10/32.5) x 1,000 = 308 feet long. Part (C) of figure A-1 shows the single 8-inch pipe which is equivalent to section ABCD shown in part (A) of figure A-1.

A-3. Alternative equivalent pipe procedure. Several variations of the equivalent pipe procedure are possible. The following is an alternative procedure for converting the pipe network of figure A-1 to a single equivalent 8-inch pipe, assuming that C = 100 for all pipes.

a. Arbitrarily select a rate of flow to be passed through both branch ACD and branch ABD. For this example, a flow of 0.5 mgd is used.

b. Calculate the losses of head through branches ACD and ABD.

Pipe	Diameter (inches)	Loss of Head Per 1,000 feet	Length (feet)	Loss of Head (feet)
AC	8	4.18	1,000	4.18
CD	6	16.90	800	13.52
AB	10	1.41	700	0.99
BD	12	0.58	2,000	1.16

Loss of head through ACD = 4.18 feet + 13.52 feet = 17.70 feet
Loss of head through ABD = 0.987 feet + 1.16 feet = 2.147 feet

c. Adjust the flow in branch ABD for the same loss of head as in branch ACD. This can be done with the following equation.

$$\frac{Q_2}{Q_1} = \left(\frac{HL_2}{HL_1}\right)^{0.54}$$

where:

Q_1 = initial flow in pipe
Q_2 = final flow in pipe
HL_1 = initial friction loss of head through the pipe
HL_2 = final friction loss of head through the pipe

Thus:

$$Q_2 = Q_1\left(\frac{HL_2}{HL_1}\right)^{0.54} = 0.5 \times \left(\frac{17.7}{2.147}\right)^{0.54}$$

Q_2 = 1.56 mgd in ABD (loss of head = 17.7 feet)

d. Find the total rate of flow through branches ABD and ACD with a loss of head of 17.7 feet in both branches. The total flow is equal to 1.56 mgd + 0.5 mgd = 2.06 mgd.

e. Determine the length of 8-inch pipe which will have a loss of head of 17.7 feet at a rate of flow of 2.06 mgd. At this rate of flow in an 8-inch pipe, the loss of head is 57.3 feet per 1,000 feet of pipe length. The total equivalent pipe length is:

Length of equivalent 8-inch pipe = (17.7/57.3) x 1,000 = 309 feet.

A-4. Hardy Cross analysis. Equivalent pipe techniques can be used for finding flows or losses of head in simple systems, but more complex networks involving multiple withdrawal points and crossover pipes require different methods of solution. The Hardy Cross method is one means of network analysis by which accurate determination of rates of flow and losses of head through a system can be computed. It involves the application of corrections to assumed values of flow or head until the system is in hydraulic balance. If flows are to be balanced, the correction factor to be applied to network flows is found by solving:

$$\Delta Q = - \frac{\Sigma H}{n \, \Sigma(H/Q)}$$

where:

ΔQ = change in percentage of flow in a particular pipe
H = loss of head in that pipe, in feet
n = 1.85

In order to use the Hardy Cross method, the following guidelines must be observed.

a. The configuration of the pipe network to be analyzed must be known or estimated. This includes pipe lengths, pipe diameters, and coefficients of roughness.

b. The locations and magnitudes of inflows and outflows to and from the system must be known or estimated.

c. Flows in either a clockwise or counterclockwise direction may be considered positive, and those in the opposite direction will be negative. For example, if clockwise flows are assumed to be positive, counterclockwise flows will be negative. The same rule also applies for values of losses of head. Thus, the terms in the numerator of the above equation will always have the appropriate sign. The term in the denominator must always have a positive value because corresponding H and Q values have the same sign, therefore H/Q is always positive.

d. The sign of the calculated correction, ΔQ, must be observed when modifying the flows in a pipe loop. Pipes appearing in more than one loop are subject to the combined corrections for the loops in which they appear. An example of the Hardy Cross analysis is shown in figure A-2 and in table A-1. Figure A-2 gives the configuration of the pipe network and inflows and withdrawals (expressed in percent) from the network. The initial percentage of flow assumptions are shown in table A-2.

Table A-1. Computations for Hardy Cross Analysis.

Trial 1

Loop Number	Pipe Number	Pipe Diam. (inches)	Pipe Length (feet)	Initial Flow (percent)	Loss of Head (feet)	$\frac{H}{Q}$	$n\Sigma\frac{H}{Q}$	ΣH	ΔQ (percent)	Adjusted Flow (percent)
I	1	10	4,000	- 25.0	- 24.7	1.0			- 1.1	- 26.1
	2	20	8,000	+ 75.0	+ 12.9	0.2			- 1.1	+ 73.9
	3	8	4,000	+ 15.0	+ 28.4	1.9			- 1.9	+ 13.1
	4	8	8,000	- 5.0	- 7.4	1.5	8.4	9.2	- 2.0	- 7.0
II	4	8	8,000	+ 5.0	+ 7.4	1.5			+ 2.0	+ 7.0
	5	8	4,000	- 20.0	- 48.4	2.4			+ 0.9	- 19.1
	6	8	3,000	+ 17.5	+ 28.3	1.6			+ 0.9	+ 18.4
	7	10	9,000	+ 5.0	+ 2.8	0.6	11.2	- 9.9	+ 0.9	+ 5.9
III	3	8	4,000	- 15.0	- 28.4	1.9			+ 1.9	- 13.1
	8	16	8,000	+ 60.0	+ 25.3	0.4			+ 0.8	+ 60.8
	9	8	2,000	- 2.5	- 0.5	0.2			+ 0.8	- 1.7
	10	10	9,000	- 2.5	- 0.8	0.3	5.2	- 4.4	+ 0.8	- 1.7

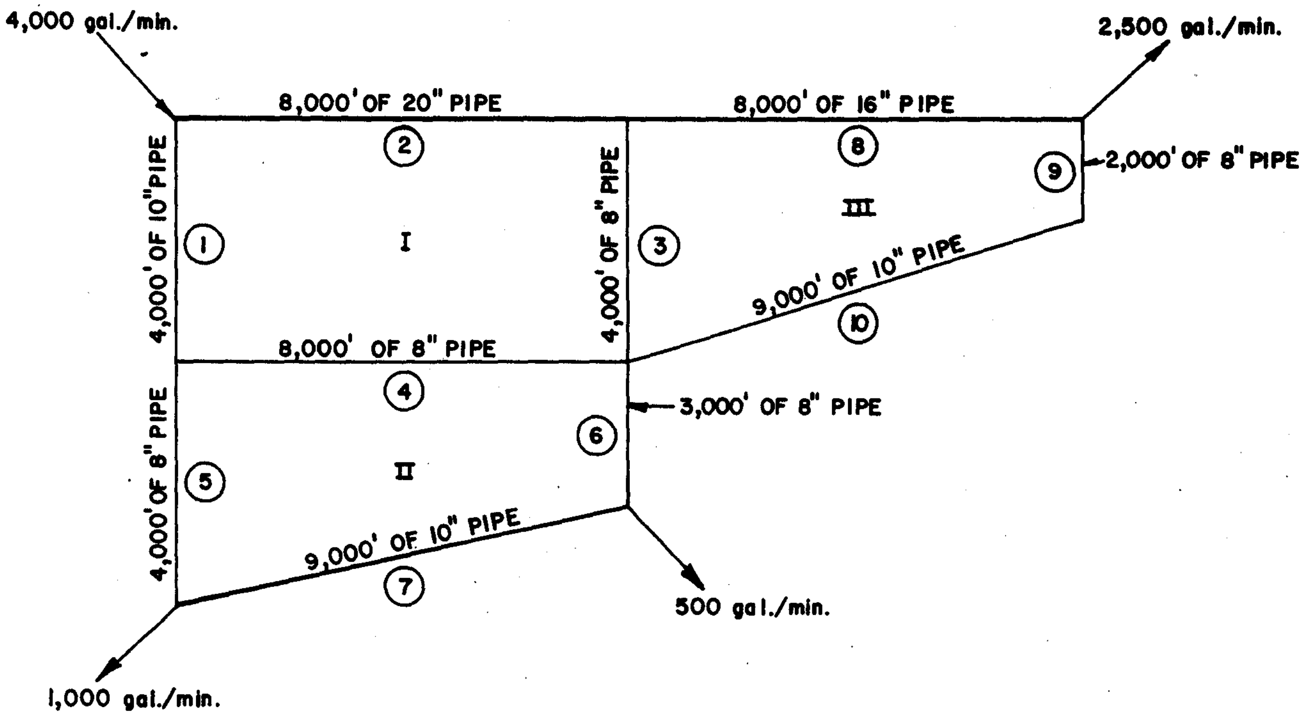
Trial 2

Loop Number	Pipe Number	Pipe Diam. (inches)	Pipe Length (feet)	Initial Flow (percent)	Loss of Head (feet)	$\frac{H}{Q}$	$n\Sigma\frac{H}{Q}$	ΣH	ΔQ (percent)	Adjusted Flow (percent)
I	1	10	4,000	- 26.1	- 26.7	1.0			+ 0.7	- 25.4
	2	20	8,000	+ 73.9	+ 12.5	0.2			+ 0.7	+ 74.6
	3	8	4,000	+ 13.1	+ 22.0	1.7			+ 1.4	+ 14.5
	4	8	8,000	- 7.0	- 13.8	2.0	9.0	- 6.0	+ 1.0	- 6.0
II	4	8	8,000	+ 7.0	+ 13.8	2.0			- 1.0	+ 6.0
	5	8	4,000	- 19.1	- 44.5	2.3			- 0.3	- 19.5
	6	8	3,000	+ 18.4	+ 31.0	1.7			- 0.3	+ 18.0
	7	10	9,000	+ 5.9	+ 3.8	0.6	12.3	+ 4.1	- 0.3	+ 5.5
III	3	8	4,000	- 13.1	- 22.0	1.7			- 1.4	- 14.5
	8	16	8,000	+ 60.8	+ 25.9	0.4			- 0.7	+ 60.1
	9	8	2,000	- 1.7	- 0.2	0.1			- 0.7	- 2.4
	10	10	9,000	- 1.7	- 0.4	0.2	4.6	+ 3.3	- 0.7	- 2.4

Trial 3

Loop Number	Pipe Number	Pipe Diam. (inches)	Pipe Length (feet)	Initial Flow (percent)	Loss of Head (feet)	$\frac{H}{Q}$	$n\Sigma\frac{H}{Q}$	ΣH	ΔQ (percent)	Adjusted Flow (percent)
I	1	10	4,000	- 25.4	- 25.4	1.0			- 0.4	- 25.8
	2	20	8,000	+ 74.6	+ 12.8	0.2			- 0.4	+ 74.2
	3	8	4,000	+ 14.5	+ 26.5	1.8			- 0.9	+ 13.6
	4	8	8,000	- 6.0	- 10.3	1.7	8.8	3.5	- 0.6	- 6.6
II	4	8	8,000	+ 6.0	+ 10.3	1.7			+ 0.6	+ 6.6
	5	8	4,000	- 19.5	- 45.9	2.4			+ 0.2	- 19.3
	6	8	3,000	+ 18.0	+ 30.0	1.7			+ 0.2	+ 18.2
	7	10	9,000	+ 5.5	+ 3.4	0.6	11.8	- 2.2	+ 0.2	+ 5.7
III	3	8	4,000	- 14.5	- 26.5	1.8			+ 0.9	- 13.6
	8	16	8,000	+ 60.1	+ 25.4	0.4			+ 0.5	+ 60.6
	9	8	2,000	- 2.4	- 0.5	0.2			+ 0.5	- 1.9
	10	10	9,000	- 2.4	- 0.7	0.3	5.1	- 2.3	+ 0.5	- 1.9

U. S. Army Corps of Engineers



U. S. Army Corps of Engineers

FIGURE A-2. EXAMPLE PIPE NETWORK FOR HARDY CROSS ANALYSIS

Table A-2. Initial Flow Assumptions

Pipe number	Flow (Percent)	Direction of flow
1	-25	Counterclockwise
2	75	Clockwise
3	15	Clockwise (Loop I)
4	-5	Counterclockwise (Loop I)
5	- 20	Counterclockwise
6	17.5	Clockwise
7	5.0	Clockwise
8	60	Clockwise
9	- 2.5	Counterclockwise
10	- 2.5	Counterclockwise

The computational procedure used in determining the actual flows is shown in table A-1. All pipes are assumed to have a roughness coefficient of 130; final flow percents and values are shown in table A-3.

Table A-3. Final Flow Values

Pipe number	Flow (percent)	Flow (gpm)	Direction of flow
1	25.8	1,033	Counterclockwise
2	74.2	2,967	Clockwise
3	13.6	544	Clockwise (Loop I)
4	6.6	263	Counterclockwise (Loop I)
5	19.3	771	Counterclockwise
6	18.2	730	Clockwise
7	5.7	224	Clockwise
8	60.6	2,423	Clockwise
9	1.9	77	Counterclockwise
10	1.9	77	Counterclockwise

A-5. Other methods of hydraulic analysis. Other hydraulic analysis techniques may be used if appropriate. Such techniques may include, but are not limited to, Newton-Raphson network analysis and network simulation with analog computers.