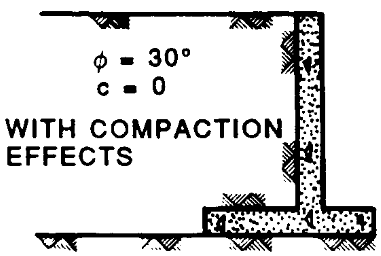
# APPENDIX M

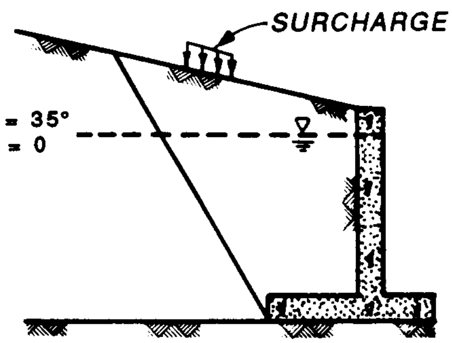
## LATERAL EARTH PRESSURE COMPUTATIONS, EXAMPLES

### SUMMARY OF EXAMPLES

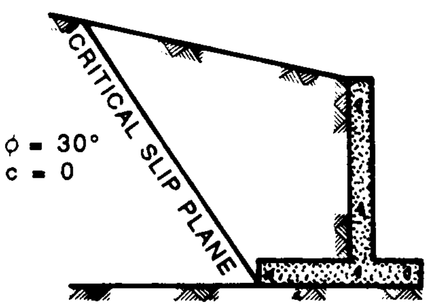
Example 1, Page M-3



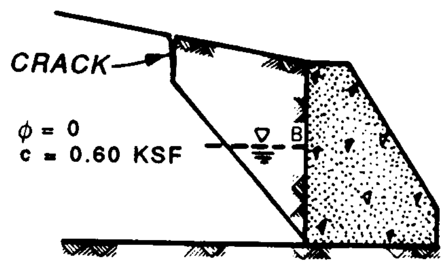

Example 4, Page M-11



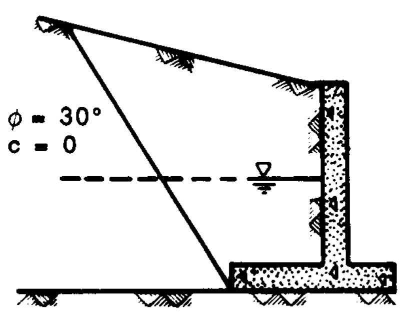

Example 2, Page M-5



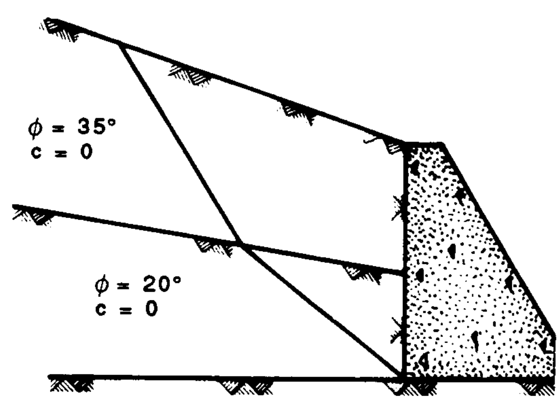

Example 5, Page M-21



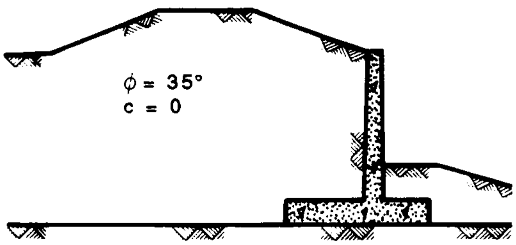

Example 3, Page M-7



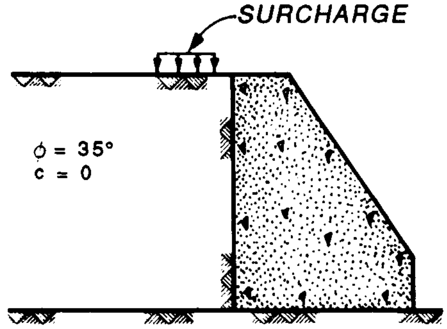

Example 6, Page M-28



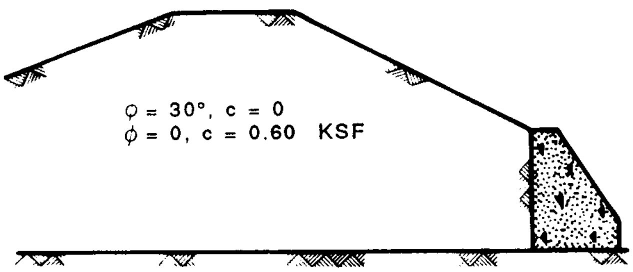
Example 7, Page M-35



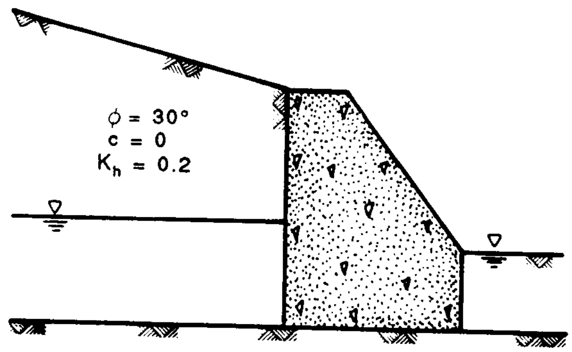

Example 10, Page M-58



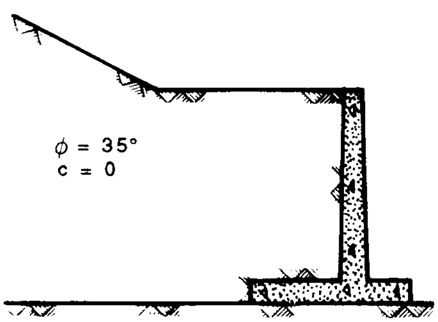

Example 8, Page M-41




Example 11, Page M-64



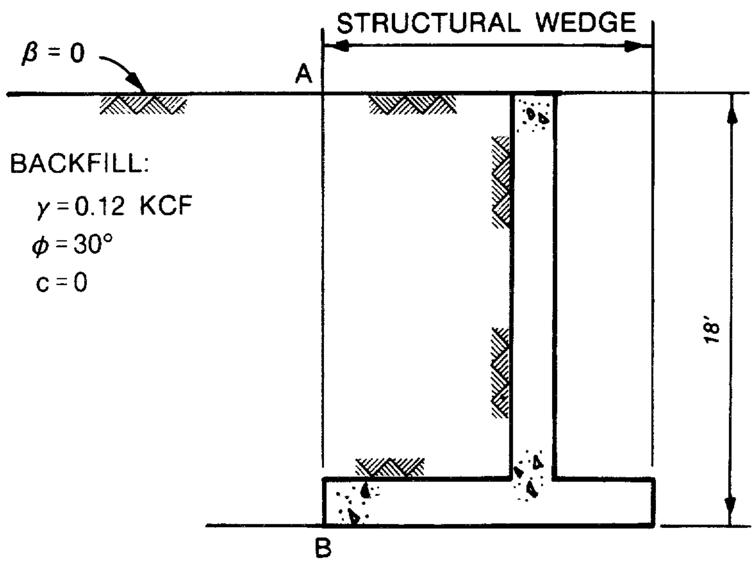

Example 9, Page M-52



LATERAL EARTH PRESSURE COMPUTATIONS, EXAMPLES

M-1. EXAMPLE 1. Find the Lateral Earth Force and its pressure distribution on Surface AB. Consider the effects of compaction in accordance with paragraph 3-17.

P = Line load for compaction roller = 5 k/ft




a. Active and passive pressure coefficients.

SMF = 1.00 , $\phi_d = \phi = 30°$

$K_A = \tan^2 (45° - \phi_d/2) = 1/3$ [3-15]

$K_P = \tan^2 (45° - \phi_d/2) = 3$ [3-20]

b. At-rest earth pressure coefficient.

SMF = 2/3 , $\phi_d = \tan^{-1} (2/3 \tan \phi) = 21°$ (from paragraph 3-11c)

$K_o = \tan^2 (45° - \phi_d/2) = 0.47$

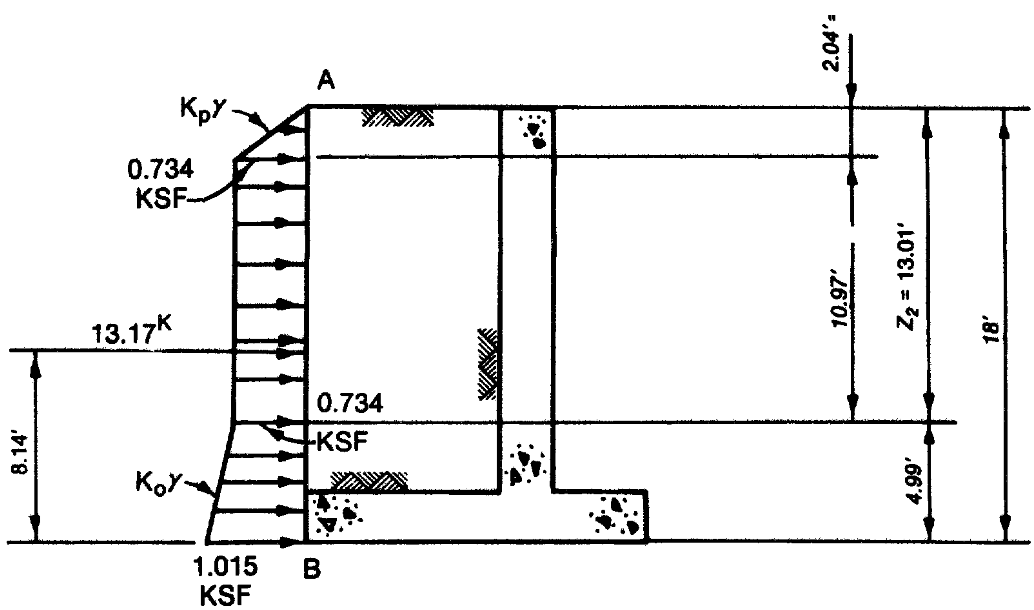
c. <u>Calculation of earth pressures</u>. From Figure 3-30:

$$z_{cr} = \sqrt{\frac{2K_AK_oP}{\pi\gamma}} = \sqrt{\frac{2/3(0.47)(5)}{\pi(0.12)}} = 2.04 \text{ ft}$$

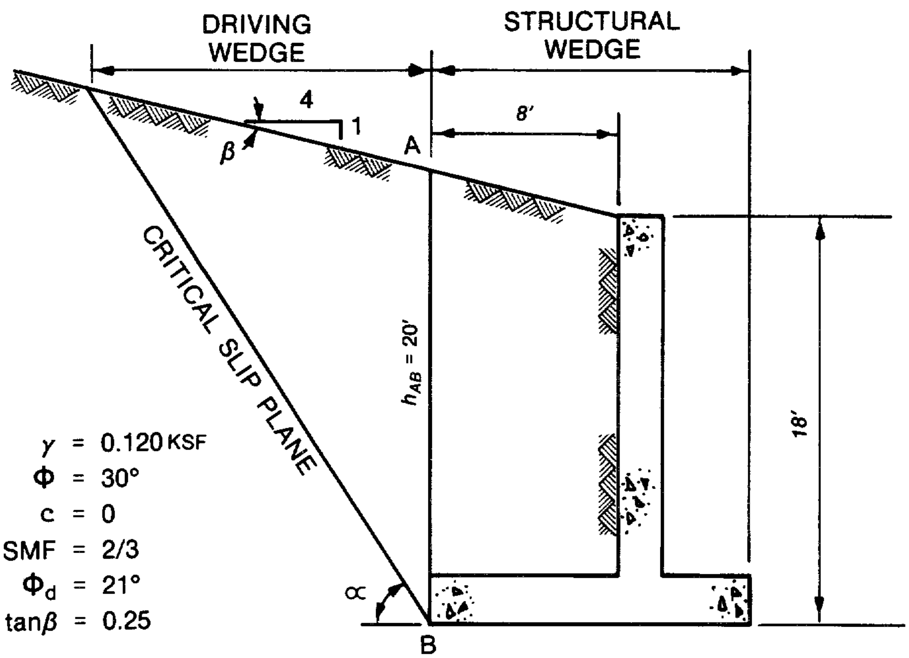
$$z_2 = \sqrt{\frac{2P}{K_AK_o\pi\gamma}} = \sqrt{\frac{2(5)}{1/3(0.47)(\pi)(0.12)}} = 13.01 \text{ ft}$$

$$P'_{hm} = \sqrt{\frac{2K_oP\gamma}{K_A\pi}} = \sqrt{\frac{2(0.47)(5)(0.12)}{(1/3)\pi}} = 0.734 \text{ ksf}$$

The force and pressure distribution are shown below:



M-2. EXAMPLE 2. Find the lateral earth force and pressure distribution acting on Surface AB.




a. Calculate α .

$$c_1 = 2 \tan \phi_d = 2(0.383864) = 0.767728 \qquad [3\text{-}26]$$

$$c_2 = 1 - \tan \phi_d \tan \beta - \frac{\tan \beta}{\tan \phi_d} \qquad [3\text{-}27]$$

$$c_2 = 1 - 0.383864(0.25) - \frac{0.25}{0.383864} = 0.252762$$

$$\alpha = \tan^{-1}\left(\frac{c_1 + \sqrt{c_1^2 + 4c_2}}{2}\right) = 45.466° \qquad [3\text{-}25]$$

b. Lateral earth pressure coefficient (see Appendix H). From the equations contained in Appendix H:

$$K_1 = \left(\frac{1 - \tan\phi_d \cot\alpha}{1 + \tan\phi_d \tan\alpha}\right)\left(\frac{\tan\alpha}{\tan\alpha - \tan\beta}\right)$$

$$K_1 = \left(\frac{1 - 0.383864 \times 0.983864}{1 + 0.383864 \times 1.016400}\right)\left(\frac{1.016400}{0.766400}\right) = 0.5937$$

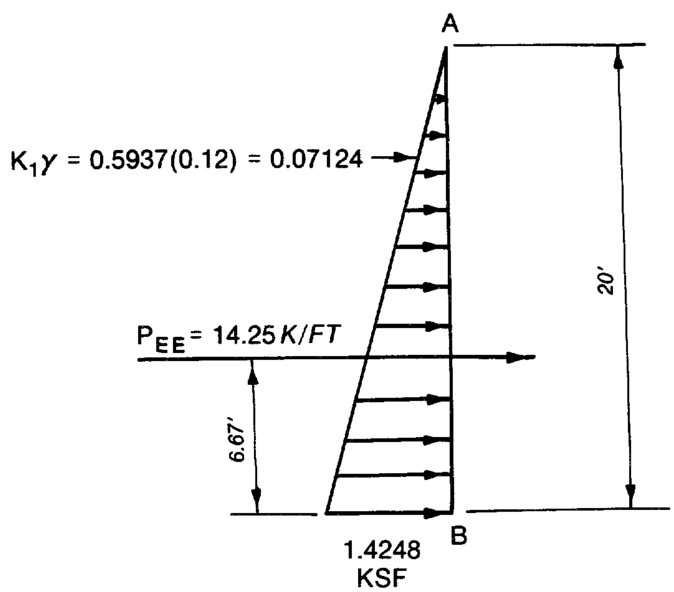
Alternatively $K_1$ may be calculated using Equation 3-14:

$$K_1 = \frac{\cos^2\phi_d}{\left[1 + \sqrt{\frac{\sin\phi_d \sin\left(\phi_d - \beta\right)}{\cos\beta}}\right]^2}$$

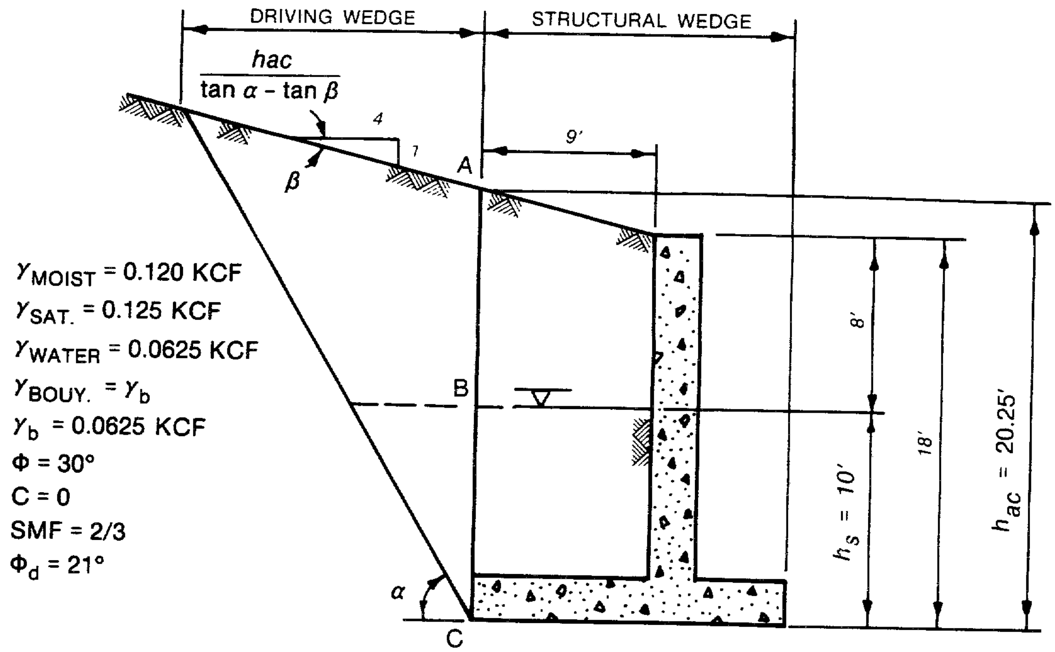
$$\beta = \tan^{-1}(0.25) = 14.0362° \ , \quad \phi_d - \beta = 6.9638°$$

$$K_1 = \frac{(0.933580)^2}{\left[1 + \sqrt{\frac{0.358368(0.121242)}{0.970143}}\right]^2} = 0.5937$$

c. Lateral force and pressure distribution. The lateral force and pressure distribution are shown in the figure below:



M-3. EXAMPLE 3. Find lateral earth and water forces acting on Surface ABC. Use earth pressure coefficients and check by iteration of Equation 3-23.




a. Lateral earth pressure coefficients (see Appendix H). The critical slip-plane angle α = 45.466° (from Example 2)

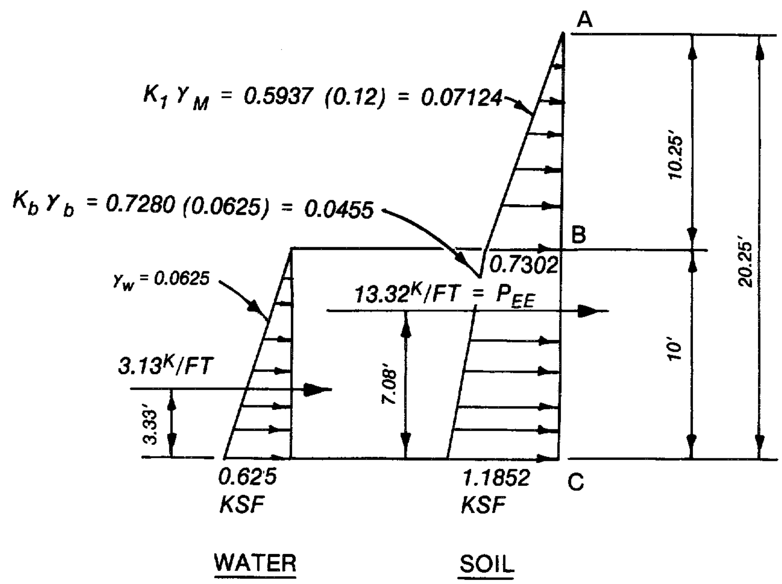
$$K = \frac{1 - \tan \phi_d \cot \alpha}{1 + \tan \phi_d \tan \alpha} = \frac{1 - 0.383864(0.983864)}{1 + 0.383864(1.016400)} = 0.447668$$

$$K_1 = K\left(\frac{\tan \alpha}{\tan \alpha - \tan \beta}\right) = 0.447668\left(\frac{1.016400}{0.766400}\right) = 0.5937$$

$$K_b = K\left[1 + \left(\frac{\tan \alpha}{\tan \alpha - \tan \beta} - 1\right)\left(\frac{\gamma_m}{\gamma_b}\right)\right] = 0.447668\left[1 + \left(\frac{1.0164}{0.7664} - 1\right)\left(\frac{0.120}{0.0625}\right)\right]$$

$$K_b = 0.7280$$

b. Lateral forces and pressure distribution. The lateral forces and pressure distribution are shown in the figure below:




c. Check soil force by iterating Equation 3-23.

W = total wedge weight, including water

$$W = \frac{\gamma_m h_{AC}^2}{2\ (\tan\alpha - \tan\beta)} + \frac{\left(\gamma_{sat} - \gamma_m\right) h_s^2}{2\ \tan\alpha}$$

$$W = \frac{0.12(20.25)^2}{2\ (\tan\alpha - \tan\beta)} + \frac{(0.005)(10)^2}{2\ \tan\alpha} = \frac{24.604}{\tan\alpha - \tan\beta} + \frac{0.25}{\tan\alpha}$$

$$U = \text{uplift} = \frac{\gamma_{water} h_s^2}{2\ \sin\alpha} = \frac{0.0625(10)^2}{2\ \sin\alpha} = \frac{3.125}{\sin\alpha}$$

$$P_w = \frac{\gamma_{water} h_s^2}{2} = \frac{0.0625(10)^2}{2} = 3.125\ \text{k/ft}$$

$$P_{EE} = \frac{W\left(\tan\alpha - \tan\phi_d\right) + U\tan\phi_d/\cos\alpha}{1 + \tan\phi_d\tan\alpha} - P_w$$

Let $\alpha = 45.466°$:

$$W = \frac{24.604}{0.766400} + \frac{0.25}{1.016400} = 32.3493 \text{ k/ft}$$

$$U = \frac{3.125}{0.712834} = 4.3839 \text{ k/ft}$$

$$P_{EE} = \frac{32.3493(1.016400 - 0.383864) + 4.3839(0.383864)/0.701332}{1 + 0.383864(1.016400)}$$

$$- 3.125 = 13.3203 \text{ k/ft}$$

Let $\alpha = 44.466°$:

$$W = \frac{24.604}{0.731531} + \frac{0.25}{0.981531} = 33.8883 \text{ k/ft}$$

$$U = \frac{3.125}{0.700486} = 4.4612 \text{ k/ft}$$

$$P_{EE} = \frac{33.8883(0.981531 - 0.383864) + 4.4612(0.383864)/0.713666}{1 + 0.383864(0.981531)}$$

$$- 3.125 = 13.3290 \text{ k/ft} > 13.3203 \text{ k/ft}$$

Let $\alpha = 43.466°$:

$$W = \frac{24.604}{0.697837} + \frac{0.25}{0.947837} = 35.5213 \text{ k/ft}$$

$$U = \frac{3.125}{0.687924} = 4.5427 \text{ k/ft}$$

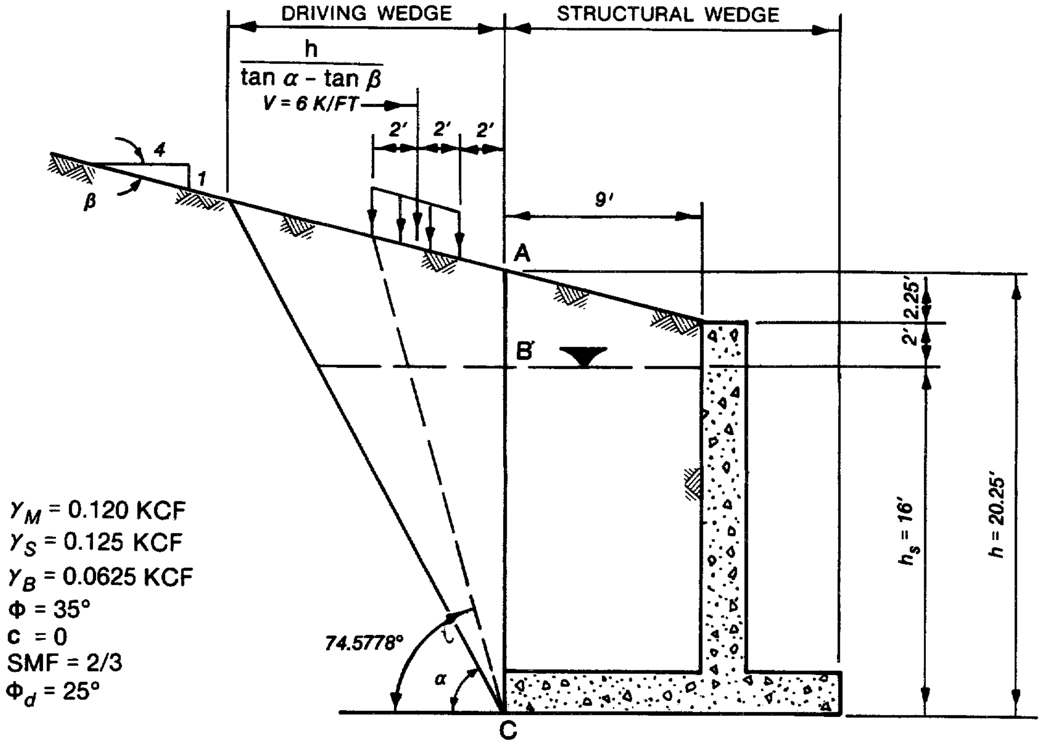
$$P_{EE} = \frac{35.5213(0.947837 - 0.383864) + 4.5427(0.383864)/0.725783}{1 + 0.383864(0.947837)}$$

$$- 3.125 = 13.3254 \text{ k/ft} < 13.3290 \text{ k/ft}$$

| |
|---|
| $\alpha$ critical = 44.466° instead of 45.466°.<br>$P_{max} = P_{EE}$ = 13.33 k/ft instead of 13.32 k/ft |

d. Conclusion. These small differences in $\alpha$ and $P_{EE}$ are due to the fact that the effect of water on the critical slip-plane angle is neglected in Equations 3-26 and 3-27. These differences are well within the permissible range of error required for soil pressure calculations. It can be concluded that for a cohensionless soil without a finite surcharge (V), it is not necessary to consider the effect of water when calculating the value of $\alpha$.

M-4. EXAMPLE 4. Find lateral soil force ($P_{EE}$) that will act on Surface ABC. Show the effect of water on slip-plane angle ($\alpha$), and earth force.




a. Calculate $\alpha$--neglecting effect of water. From Equation 3-30 (omitting the c term)

$$A = \tan \phi_d - \frac{2V(1 + \tan^2 \phi_d)}{\gamma_m h^2} = 0.466308 - \frac{2(6)(1.217443)}{0.12(20.25)^2}$$

A = 0.169416

From Equation 3-28 (omitting the c term)

$$c_1 = \frac{2 \tan^2 \phi_d - \dfrac{4V \tan \beta \left(1 + \tan^2 \phi_d\right)}{\gamma_m h^2}}{A}$$

$$= \frac{2(0.466308)^2 - \dfrac{4(6)(1/4)(1.217443)}{0.12(20.25)^2}}{0.169416}$$

$$c_1 = 1.690751$$

From Equation 3-29 (omitting the $c$ term)

$$c_2 = \frac{\tan \phi_d (1 - \tan \phi_d \tan \beta) - \tan \beta + \dfrac{2V \tan^2 \beta \left(1 + \tan^2 \phi_d\right)}{\gamma_m h^2}}{A}$$

$$c_2 = \frac{0.466308\left[(1 - 0.466308(1/4)\right] - \frac{1}{4} + \dfrac{2(6)(1/16)(1.217443)}{0.12(20.25)^2}}{0.169416} = 1.065442$$

$$\alpha = \tan^{-1}\left(\frac{c_1 + \sqrt{c_1^2 + 4c_2}}{2}\right) = \underline{\underline{65.354°}} \qquad [3\text{-}25]$$

b. <u>Calculate $\alpha$--include effect of water</u>. Use $\gamma_{avg}$ instead of $\gamma_m$, where:

$$\gamma_{avg} = \left[\frac{\gamma_m h^2}{2\ (\tan \alpha - \tan \beta)} - \frac{\left(\gamma_m - \gamma_b\right) h_s^2}{2 \tan \alpha}\right] \div \left[\frac{h^2}{2\ (\tan \alpha - \tan \beta)}\right]$$

Let $\alpha = 74.5778°$

$$\gamma_{avg} = \left[\frac{0.12(20.25)^2}{2(3.374990)} - \frac{0.0575(16)^2}{2(3.624990)}\right] \div \left[\frac{(20.25)^2}{2(3.374990)}\right]$$

$$\gamma_{avg} = 0.087 \text{ kcf}$$

$$A = 0.466308 - \frac{2(6)(1.217443)}{0.087(20.25)^2} = 0.056802$$

$$c_1 = \frac{2(0.466308)^2 - \dfrac{4(6)(1/4)(1.217443)}{0.087(20.25)^2}}{0.056802} = 4.051497$$

$$c_2 = \frac{0.466308\left[1 - 0.466308(1/4)\right] - \dfrac{1}{4} + \dfrac{2(6)(1/16)(1.217443)}{0.087(20.25)^2}}{0.056802}$$

$$c_2 = 3.301668$$

$$\alpha = \tan^{-1}\left(\frac{c_1 + \sqrt{c_1^2 + 4c_2}}{2}\right) = \underline{\underline{78.104°}} > 74.5778°$$

Use: $\alpha = 74.5778°$ , the entire surcharge will not lie on the top surface of the wedge when the angle is greater.

c. Calculate earth pressure coefficients (see Appendix H). For $\alpha = 65.354°$ (neglecting effect of water):

$$K = \frac{1 + \tan \phi_d \cot \alpha}{1 - \tan \phi_d \tan \alpha}$$

$$K = \frac{1 - 0.466308(0.458807)}{1 + 0.466308(2.179565)} = 0.389841$$

$$K_1 = K\left(\frac{\tan \alpha}{\tan \alpha - \tan \beta}\right)$$

$$K_1 = 0.389841\left(\frac{2.179565}{1.929565}\right) = 0.44035$$

$$K_b = K\left[1 + \left(\frac{\tan\alpha}{\tan\alpha - \tan\beta} - 1\right)\left(\frac{\gamma_m}{\gamma_b}\right)\right]$$

$$K_b = 0.389841\left[1 + \left(\frac{2.179565}{1.929565} - 1\right)\left(\frac{0.120}{0.0625}\right)\right] = 0.48682$$

$$K_v = K \tan\alpha$$

$$K_v = 0.389841(2.179565) = 0.84968$$

For $\alpha = 74.5778°$ (including effect of water):

$$K = \frac{1 - 0.466308(0.275863)}{1 + 0.466308(3.624990)} = 0.323883$$

$$K_1 = 0.323883\left(\frac{3.624990}{3.374990}\right) = 0.34787$$

$$K_b = 0.323883\left[1 + \left(\frac{3.624990}{3.374990} - 1\right)\left(\frac{0.120}{0.0625}\right)\right] = 0.36995$$

$$K_v = 0.323883(3.624990) = 1.17407$$

d. Calculate lateral soil force (using coefficients). For $\alpha = 65.354°$:

$$P_\gamma = (1/2)K_1\gamma_m\left(h - h_s\right)^2 + (1/2)(h_s)\left[2K_1\gamma_m(h - h_s) + K_b\gamma_b h_s\right]$$

$$P_\gamma = (1/2)(0.44035)(0.12)(4.25)^2 + (1/2)(16)\Big[2(0.44035)(0.12)(4.25) + 0.48682(0.0625)(16)\Big]$$

$$P_\gamma = 7.965 \text{ k/ft}$$

$$P_v = K_v V = 0.84968(6) = 5.098 \text{ k/ft}$$

$$P_{EE} = 7.965 + 5.098 = \underline{\underline{13.063 \text{ k/ft}}}$$

For $\alpha = 74.5778°$:

$$P_\gamma = (1/2)(0.34787)(0.12)(4.25)^2 + (1/2)(16)\Big[2(0.34787)(0.12)(4.25) + 0.36995(0.0625)(16)\Big]$$

$$P_\gamma = 6.175 \text{ k/ft}$$

$$P_v = 1.17407(6) = 7.044 \text{ k/ft}$$

$$P_{EE} = 6.175 + 7.044 = \underline{\underline{13.219 \text{ k/ft}}}$$

e. Find lateral soil force by iteration of Equation 3-23. Simplifying Equation 3-23.

$$P_{EE} = \frac{(W + V)\ (\tan\alpha - \tan\phi_d) + U \tan\phi_d/\cos\alpha}{1 + \tan\phi_d \tan\alpha} - P_w$$

$$W = \frac{\gamma_m h^2}{2\ (\tan\alpha - \tan\beta)} + \frac{\left(\gamma_s - \gamma_m\right) h_s^2}{2 \tan\alpha}$$

$$W = \frac{0.12(20.25)^2}{2\ (\tan\alpha - \tan\beta)} + \frac{(0.005)(16)^2}{2 \tan\alpha} = \frac{24.6038}{\tan\alpha - \tan\beta} + \frac{0.64}{\tan\alpha}$$

$$U = \frac{\gamma_w h_s^2}{2 \sin\alpha} = \frac{0.0625(16)^2}{2 \sin\alpha} = \frac{8}{\sin\alpha}$$

$$P_w = \frac{\gamma_w h_s^2}{2} = \frac{0.0625(16)^2}{2} = 8.000 \text{ k/ft}$$

Let $\alpha = 73.5778°$

$$W = \frac{24.6038}{3.142854} + \frac{0.64}{3.392854} = 8.0171 \text{ k/ft}$$

$$U = \frac{8.00}{0.959205} = 8.3402 \text{ k/ft} \quad , \quad W + V = 14.0171 \text{ k/ft}$$

$$P_{EE} = \frac{14.0171(3.392854 - 0.466308) + 8.3402(0.466308)/0.282713}{1 + 0.466308(3.392854)}$$

$$- 8.000 = \underline{\underline{13.214 \text{ k/ft}}}$$

Let $\alpha = 74.5778°$

$$W = \frac{24.6038}{3.374990} + \frac{0.64}{3.624990} = 7.4666 \text{ k/ft}$$

$$W + V = 13.4666 \text{ k/ft} \quad , \quad U = \frac{8.00}{0.963992} = 8.2988 \text{ k/ft}$$

$$P_{EE} = \frac{13.4666(3.624990 - 0.466308) + 8.2988(0.466308)/0.265930}{1 + 0.466308(3.624990)}$$

$$- 8.000 = \underline{\underline{13.220 \text{ k/ft}}} > 13.214$$

Let $\alpha = 75.5778°$ (all of $V$ does not lie on top of wedge)

$$\frac{h}{\tan \alpha - \tan \beta} = \frac{20.25}{3.638487} = 5.5655 \text{ ft}$$

$$V = \left(\frac{5.5655 - 2}{4}\right) 6 \text{ k/ft} = 5.3483 \text{ k/ft}$$

$$W = \frac{24.6038}{3.638487} + \frac{0.64}{3.888487} = 6.9267 \text{ k/ft}$$

$$W + V = 12.2750 \text{ k/ft} , \quad U = 8/0.96847 = 8.2603$$

$$P_{EE} = \frac{12.275(3.888487 - 0.466308) + 8.2603(0.466308)/0.249065}{1 + 0.466308(3.888487)}$$

$$- 8.000 = \underline{\underline{12.429 \text{ k/ft}}} < 13.220$$

$P_{EE} = \underline{\underline{13.220 \text{ k/ft}}}$ (agrees with coefficient solution where $\alpha$ is found considering the effect of water)

f. <u>Determine line of action for lateral force due to V using the approximate method of Figure 3-29</u>. Find $\alpha'$, the slip-plane angle without surcharge:

Use SMF = 1.00 in Equations 3-26 and 3-27.

$$c_1 = 2 \tan \phi = 2(0.700208) = 1.400416$$

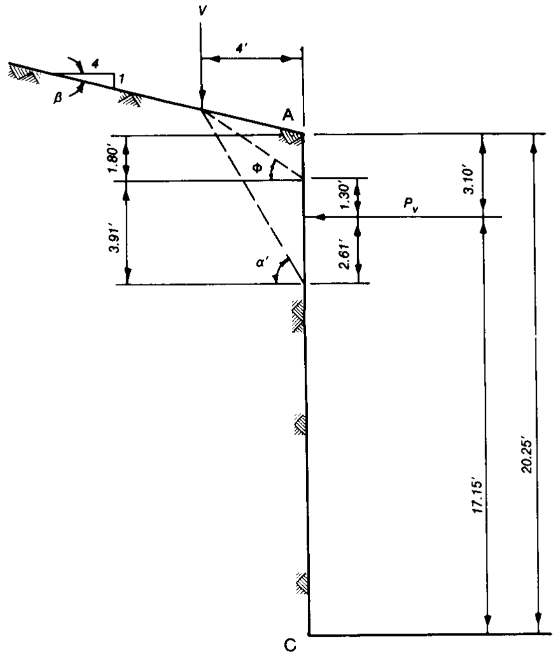
$$c_2 = 1 - \tan \phi \tan \beta - \frac{\tan \beta}{\tan \phi}$$

$$c_2 = 1 - 0.700208(1/4) - \frac{(1/4)}{0.700208} = 0.467911$$

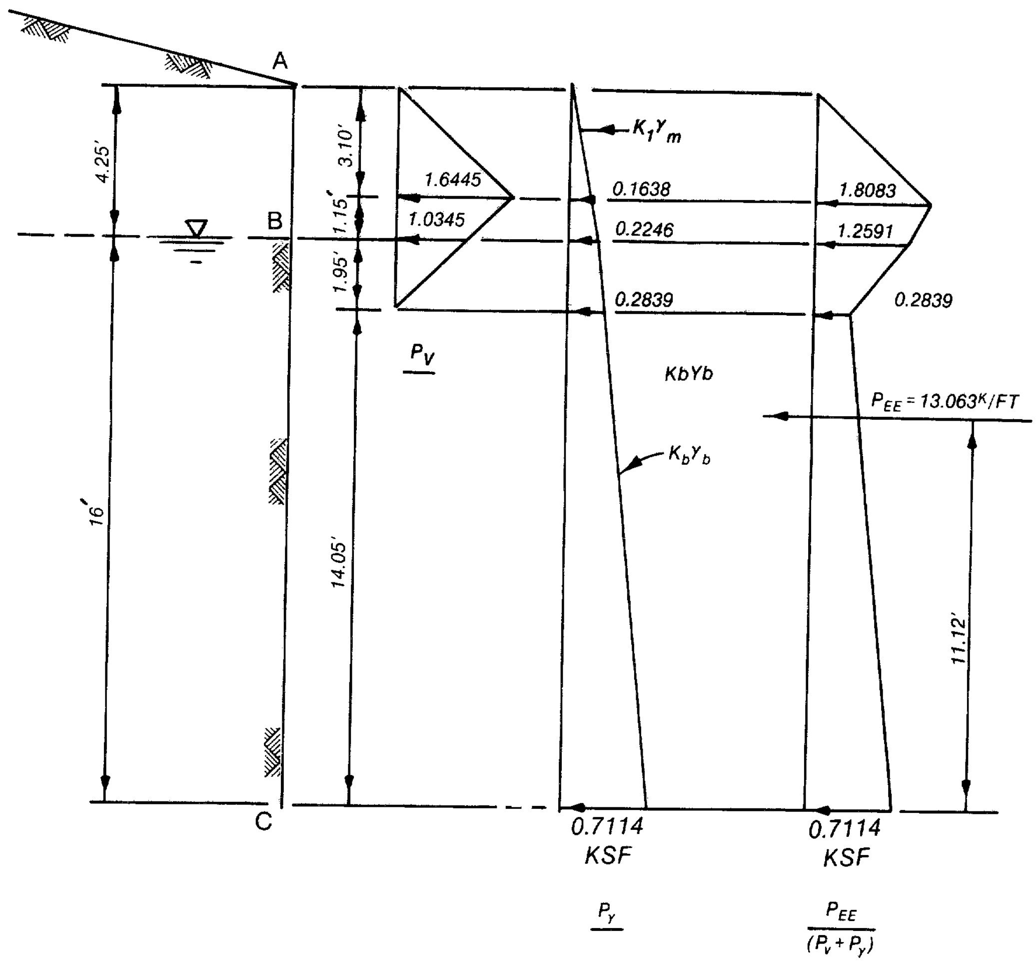
From Equation 3-25

$$\alpha' = \tan^{-1}\left(\frac{c_1 + \sqrt{c_1^2 + 4c_2}}{2}\right) = 59.22°$$

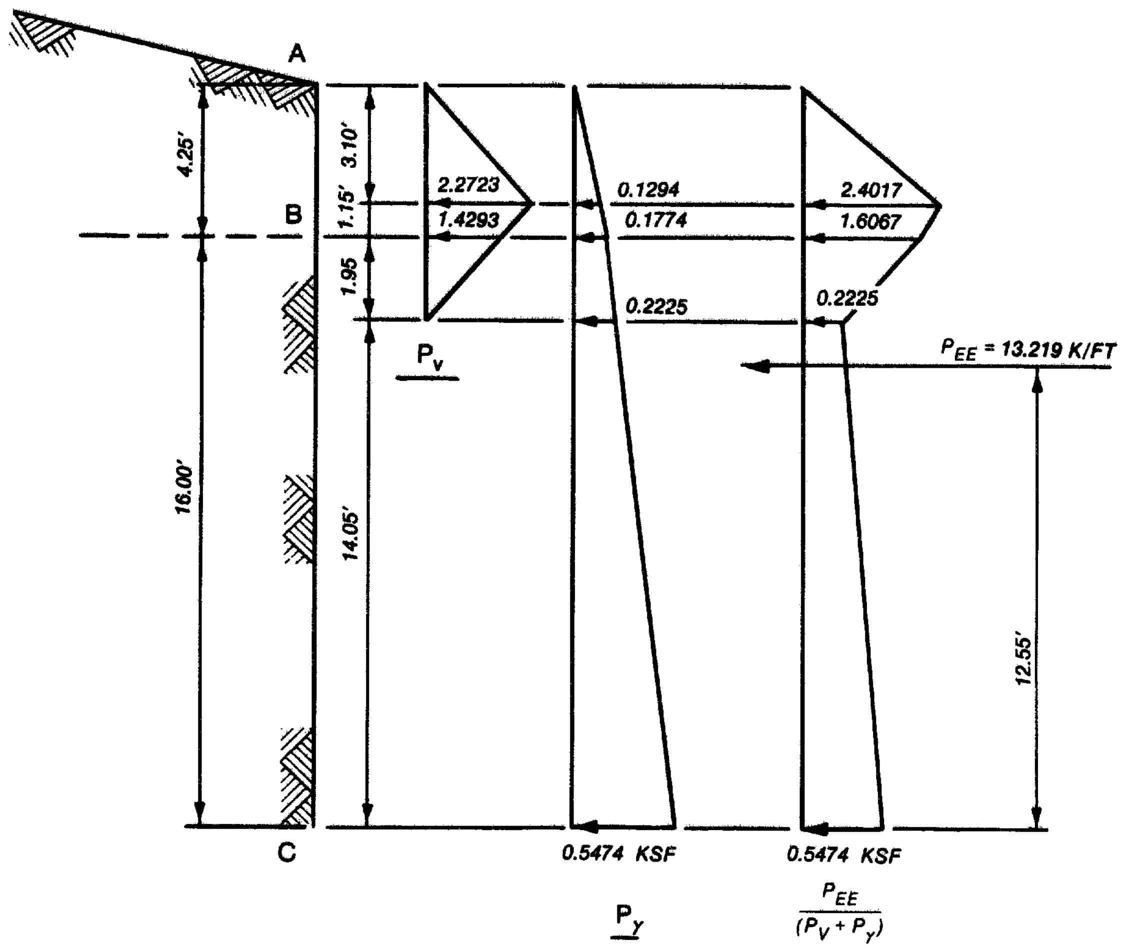
The location of $P_V$ is determined in the following figure.




g. Pressure distribution and line of action of earth force $P_{EE}$ for $\alpha = 65.354°$. The pressure distribution and line of action of earth force $P_{EE}$ for $\alpha = 65.354°$ is shown on the following page. A pressure distribution must be assumed for the lateral force due to V since only the location of the resultant force is known. The pressure distribution will be defined by an isosceles triangle with the apex located at the point of application of the resultant force due to V . See example 10 of this appendix for the computation of the lateral force due to a surcharge by the elastic method.

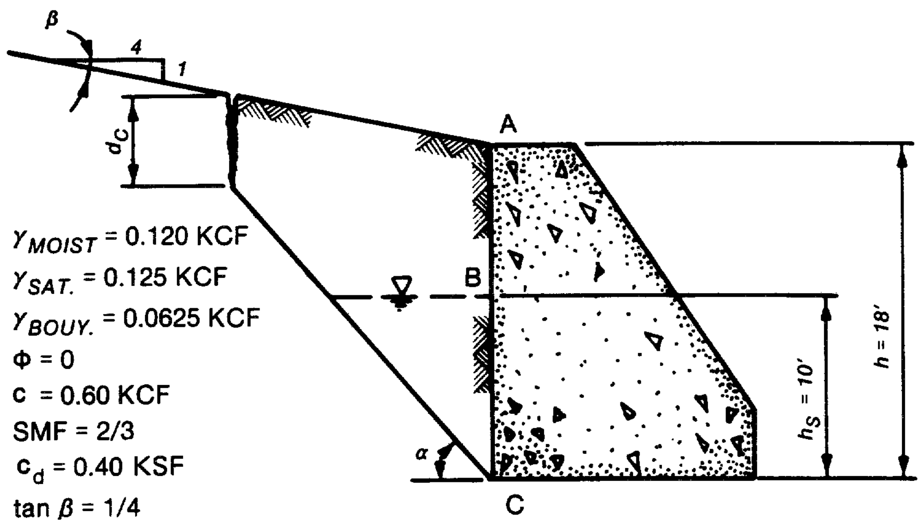



h. <u>Pressure distribution and line of action of earth force $P_{EE}$ for $\alpha = 74.5778°$</u>. The pressure distribution and line of action of earth force $P_{EE}$ for $\alpha = 74.5778°$ is shown on the following page:




i. Conclusion. For a cohensionless soil when calculating $\alpha$ from Equations 3-28, 3-29 and 3-30, the unit weight of soil ($\gamma$) in the strip surcharge (V) term should be the average effective unit weight ($\gamma_{avg}$). $\gamma_{avg}$ should be calculated using the moist unit weight above the water table and the buoyant unit weight below the water table.

M-5. EXAMPLE 5. Find lateral earth force acting on Surface ABC. Use moist unit weight of soil to calculate $\alpha$ . Check solution by iteration of Equation 3-23.




a. Calculate slip-plane angle $\alpha$. Estimate $\alpha = 40°$:

Using Equation I-2 with $\phi = 0$ yields

$$d_c = \frac{c_d/\gamma_m}{\sin \alpha \cos \alpha} = \frac{0.4/0.12}{0.642788(0.766044)} = 6.77 \text{ ft} \quad \text{(Appendix I)}$$

From Equation 3-30

$$A = \frac{2c_d}{\gamma_m(h + d_c)} = \frac{2(0.4)}{0.12(24.77)} = 0.269143$$

From Equation 3-28

$$c_1 = \frac{\dfrac{4c_d \tan \beta}{\gamma_m(h + d_c)}}{A} = \frac{\dfrac{4(0.4)(1/4)}{0.12(24.77)}}{0.269143} = 0.50$$

From Equation 3-29

$$c_2 = \frac{-\tan\beta + \dfrac{2c_d}{\gamma_m(h + d_c)}}{A} = \frac{-\dfrac{1}{4} + \dfrac{2(0.4)}{0.12(24.77)}}{0.269143} = 0.071125$$

From Equation 3-25

$$\alpha = \tan^{-1}\left(\frac{c_1 + \sqrt{c_1^2 + 4c_2}}{2}\right) = 31.61° \neq 40°$$

Let $\alpha = 31.61°$:

$$d_c = \frac{0.4/0.12}{0.524135(0.851635)} = 7.47 \text{ ft}$$

$$A = \frac{2(0.4)}{0.12(25.47)} = 0.261746$$

$$c_1 = \frac{\dfrac{4(0.4)(1/4)}{0.12(25.47)}}{0.261746} = 0.50$$

$$c_2 = \frac{-\dfrac{1}{4} + \dfrac{2(0.4)}{0.12(25.47)}}{0.261746} = 0.044875$$

$$\alpha = \tan^{-1}\left(\frac{c_1 + \sqrt{c_1^2 + 4c_2}}{2}\right) = 30.01° \neq 31.61°$$

Let $\alpha = 30°$:

$$d_c = \frac{0.4/0.12}{0.5(0.866025)} = 7.70 \text{ ft}$$

$$A = \frac{2(0.4)}{0.12(25.70)} = 0.259403$$

$$c_1 = 0.50 \ , \quad c_2 = \frac{-\frac{1}{4} + 0.259403}{0.259403} = 0.036249$$

$$\alpha = \tan^{-1}\left(\frac{c_1 + \sqrt{c_1^2 + 4c_2}}{2}\right) = 29.43° \neq 30°$$

Let $\alpha = 29°$:

$$d_c = \frac{0.4/0.12}{0.484810(0.874620)} = 7.86 \text{ ft}$$

$$A = \frac{2(0.4)}{0.12(25.86)} = 0.257798$$

$$c_1 = 0.5 \ , \quad c_2 = \frac{-\frac{1}{4} + 0.257798}{0.257798} = 0.030248$$

$$\alpha = \tan^{-1}\left(\frac{c_1 + \sqrt{c_1^2 + 4c_2}}{2}\right) = 29.01°$$

$\underline{\underline{\alpha = 29°}}$ (nearest degree), $d_c = \underline{\underline{7.86 \text{ ft}}}$

b. Earth pressure coefficients and earth force for $\alpha = 29°$ (see Appendix H).

$$K = \frac{1 - \tan \phi_d \cot \alpha}{1 + \tan \phi_d \tan \alpha} = 1.00$$

$$K_1 = K\left(\frac{\tan \alpha}{\tan \alpha - \tan \beta}\right) = 1.00\left(\frac{0.554309}{0.304309}\right) = 1.8215$$

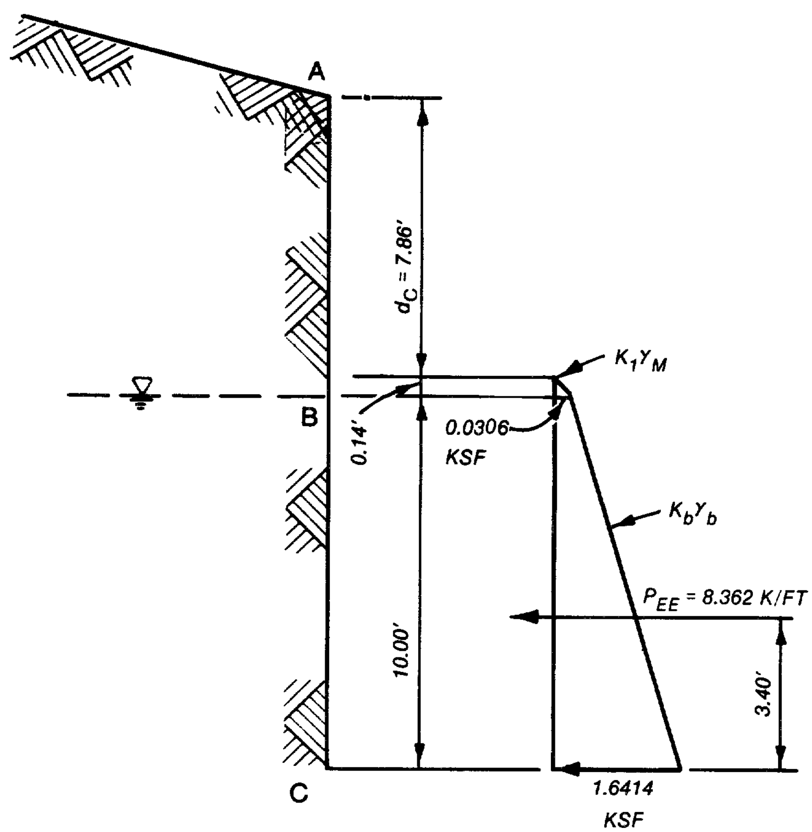
$$K_b = K\left[1 + \left(\frac{\tan\alpha}{\tan\alpha - \tan\beta} - 1\right)\left(\frac{\gamma_m}{\gamma_b}\right)\right]$$

$$K_b = 1.00\left[1 + \left(\frac{0.554309}{0.304309} - 1\right)\frac{0.120}{0.0625}\right] = 2.5773$$

$$P_{EE} = (1/2)K_1\gamma_m\left(h - h_s - d_c\right)^2 + (1/2)h_s\left[2K_1\gamma_m(h - h_s - d_c) + K_b\gamma_b h_s\right]$$

$$h = 18 \text{ ft}, \quad h_s = 10 \text{ ft}, \quad d_c = 7.86 \text{ ft}$$

$$P_{EE} = (1/2)(1.8215)(0.12)(0.14)^2 + (1/2)(10)\left[2(1.8215)(0.12)(0.14) + 2.5773(0.0625)(10)\right] = \underline{\underline{8.362 \text{ k/ft}}}$$



c. Check solution by iteration of Equation 3-23.

$$W = \frac{\gamma_m\left(h^2 - d_c^2\right)}{2(\tan\alpha - \tan\beta)} + \frac{\left(\gamma_s - \gamma_m\right)h_s^2}{2\tan\alpha}$$

$$W = \frac{0.06\left(h^2 - d_c^2\right)}{\tan\alpha - \tan\beta} + \frac{0.25}{\tan\alpha}$$

$$d_c = \frac{c_d/\gamma_m}{\sin\alpha\cos\alpha} = \frac{3.3333}{\sin\alpha\cos\alpha}$$

$$L = \frac{h - d_c}{\cos\alpha\,(\tan\alpha - \tan\beta)}\ , \quad P_w = \frac{1}{2}\gamma_w h_s^2 = 3.125\ \text{k/ft}$$

From Equation 3-23

$$P_{EE} = W\tan\alpha - c_d\, L/\cos\alpha - P_w$$

Let $\alpha = 28°$:

$$d_c = \frac{3.3333}{0.469472(0.882948)} = 8.04\ \text{ft},\quad h^2 - d_c^2 = 259.3584$$

$$L = \frac{9.96}{0.882948(0.281709)} = 40.0427\ \text{ft}$$

$$W = \frac{0.06(259.3584)}{0.281709} + \frac{0.25}{0.531709} = 55.7098\ \text{k/ft}$$

$$P_{EE} = 55.7098(0.531709) - 0.4(40.0427)/0.882948 - 3.125$$

$$P_{EE} = 8.356\ \text{k/ft}$$

Let $\alpha = 29°$:

$$d_c = \frac{3.3333}{0.484810(0.874620)} = 7.86 \text{ ft}$$

$$h^2 - d_c^2 = 262.2204 \ , \quad L = \frac{10.14}{0.87462(0.304309)} = 38.0981 \text{ ft}$$

$$W = \frac{0.06(262.2204)}{0.304309} + \frac{0.25}{0.554309} = 52.1525 \text{ k/ft}$$

$$P_{EE} = 52.1525(0.554309) - 0.4(38.0981)/0.87462 - 3.125$$

$$P_{EE} = \underline{\underline{8.360 \text{ k/ft}}} > 8.356$$

Let $\alpha = 30°$:

$$d_c = \frac{3.3333}{0.5(0.866025)} = 7.70 \text{ ft}, \quad h^2 - d_c^2 = 264.71$$

$$L = \frac{10.30}{0.866025(0.327350)} = 36.3324 \text{ ft}$$

$$W = \frac{0.06(264.71)}{0.327350} + \frac{0.25}{0.57735} = 48.9517 \text{ k/ft}$$

$$P_{EE} = 48.9517(0.57735) - 0.4(36.3324)/0.866025 - 3.125$$

$$P_{EE} = 8.356 \text{ k/ft} < 8.360$$

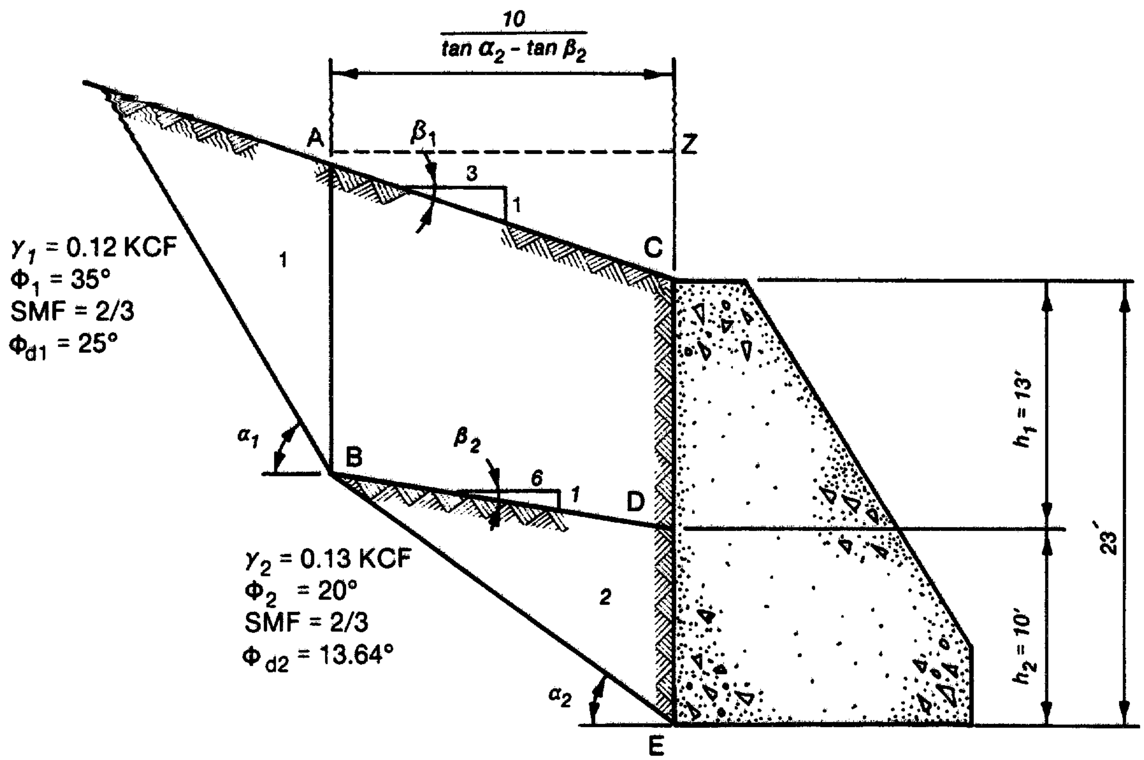
Iteration of Equation 3-23 shows that:

$\underline{\underline{\alpha_{crit} = 29°}}$ (to nearest degree), $\underline{\underline{P_{EE} = 8.360 \text{ k/ft}}}$

which agrees with the pressure coefficient solution.

d. Conclusion. Use the moist unit weight of soil ($\gamma_m$) in the cohesion terms of Equations 3-28, 3-29, and 3-24; even if the soil is partially saturated.

M-6. EXAMPLE 6. Find the lateral earth forces on Surfaces CD and DE (stratified soil).




a. Calculate $\alpha_2$ (see Appendix G). Using Equation G-25 from Appendix G

$$\gamma' = \frac{2\gamma_1 h_1}{h_2} + \gamma_2 + \frac{2\gamma_1\left(\tan\beta_1 - \tan\beta_2\right)}{\tan\alpha_2 - \tan\beta_2}$$

$$\gamma' = \frac{2(0.12)(13)}{10} + 0.13 + \frac{2(0.12)\left(\frac{1}{3} - \frac{1}{6}\right)}{\tan\alpha_2 - \frac{1}{6}} = 0.442 + \frac{0.04}{\tan\alpha_2 - \frac{1}{6}}$$

The weight of the surcharge in triangle ACZ is calculated by using Equation G-26.

$$V_\alpha = -\frac{\gamma_1 h_2^2 \left(\tan \beta_1 - \tan \beta_2\right)}{2\left(\tan \alpha_2 - \tan \beta_2\right)^2} = \frac{-0.12(10)^2\left(\frac{1}{6}\right)}{2\left(\tan \alpha_2 - \frac{1}{6}\right)^2} = \frac{-1}{\left(\tan \alpha_2 - \frac{1}{6}\right)^2}$$

First trial: $\alpha_2 = 39°$ , $\gamma' = 0.5042$ , $V_\alpha = -2.4178$

From Equation G-27

$$A' = \tan \phi_{d2} - \frac{2V_\alpha\left(1 + \tan^2 \phi_{d2}\right)}{\gamma' h_2^2}$$

$$A' = 0.242665 - \frac{2(-2.4178)(1.058886)}{0.5042(10)^2} = 0.344219$$

From Equation G-28

$$c_1' = \frac{2 \tan^2 \phi_{d2} - \dfrac{4V_\alpha \tan \beta_2 \left(1 + \tan^2 \phi_{d2}\right)}{\gamma' h_2^2}}{A'}$$

$$c_1' = \frac{2(0.242665)^2 - \dfrac{4(-2.4178)\left(\frac{1}{6}\right)(1.058886)}{0.5042(10)^2}}{0.344219} = 0.440486$$

From Equation G-29

$$c_2' = \frac{\tan \phi_{d2}\left(1 - \tan \phi_{d2} \tan \beta_2\right) - \tan \beta_2 + \dfrac{2V_\alpha \tan^2 \beta_2\left(1 + \tan^2 \phi_{d2}\right)}{\gamma' h_2^2}}{A'}$$

$$c_2' = \frac{0.242665\left(1 - 0.242665 \times \frac{1}{6}\right) - \frac{1}{6} + \frac{2(-2.4178)\left(\frac{1}{36}\right)(1.058886)}{0.5042(10)^2}}{0.344219}$$

$$c_2' = 0.184077$$

$$\alpha_2 = \tan^{-1}\left(\frac{c_1' + \sqrt{c_1'^2 + 4c_2'}}{2}\right) \qquad [G-30]$$

$$\alpha_2 = 35.1° \neq 39°$$

Second trial: $\alpha_2 = 33°$ , $\gamma' = 0.5249$ , $V_\alpha = -4.2911$

$$A' = 0.242665 - \frac{0.021178(-4.2911)}{0.5249} = 0.415797$$

$$c_1' = \frac{0.117773 - \frac{0.007059(-4.2911)}{0.5249}}{0.415797} = 0.422035$$

$$c_2' = \frac{0.066814 + \frac{0.000588(-4.2911)}{0.5249}}{0.415797} = 0.149128$$

$$\underline{\underline{\alpha_2 = 33°}}$$

b. Calculate $\alpha_1$ .

$$c_1 = 2 \tan \phi_{d1} = 2(0.466308) = 0.932616 \qquad [3-26]$$

$$c_2 = 1 - \tan \phi_{d1} \tan \beta - \frac{\tan \beta}{\tan \phi_{d1}} \qquad [3-27]$$

$$c_2 = 1 - 0.466308(1/3) - \frac{(1/3)}{0.466308} = 0.129729$$

$$\alpha_1 = \underline{\underline{46.547°}}$$

c. <u>Calculate lateral earth pressure coefficients (see Appendix H)</u>.

**Wedge 1:**

$$K = \frac{1 - 0.466308(0.947407)}{1 + 0.466308(1.055513)} = 0.374091$$

$$K_1 = 0.374091\left(\frac{1.055513}{0.722180}\right) = 0.5468$$

**Wedge 2:**

$$K = \frac{1 - 0.242665(1.539865)}{1 + 0.242665(0.649408)} = 0.541063$$

$$K_1 = 0.541063\left(\frac{0.649408}{0.482741}\right) = 0.7279$$

$$K_v = 0.541063(0.649408) = 0.3514$$

d. <u>Calculate lateral earth forces and pressures</u>.

**Wedge 1:**

$$h = h_{AB} = h_1 + \frac{h_2\ (\tan\beta_1 - \tan\beta_2)}{(\tan\alpha_2 - \tan\beta_2)}$$

$$h_{AB} = 13 + \frac{(10)\left(\frac{1}{3} - \frac{1}{6}\right)}{0.649408 - 0.166667} = 16.452 \text{ ft}$$

$$P_{EE1} = \frac{1}{2} K_1 \gamma_1 h^2$$

$$P_{EE1} = \frac{1}{2}(0.5468)(0.12)(16.452)^2 = \underline{\underline{8.88 \text{ k/ft}}}$$

$$p_{HB} = K_1 \gamma_1 h_{AB} = 0.5468(0.12)(16.452) = 1.0795$$

Wedge 2:

Treat weight of ABCD as a strip surcharge on Surface BD.

$$V = \frac{1}{2}\,(h_1 + h_{AB})(\overline{AZ})(\gamma_1)$$

$$V = \frac{1}{2}\,(13 + 16.452)(20.715)(0.12) = 36.606 \text{ k/ft}$$

$$P_\gamma = \frac{1}{2} K_1 \gamma_2 h_2^2$$

$$P_\gamma = \frac{1}{2}\,(0.7279)(0.13)(10)^2 = 4.73 \text{ k/ft}$$

$$P_V = K_v V$$

$$P_V = 0.3514(36.606) = 12.86 \text{ k/ft}$$

$$P_{EE2} = P_\gamma + P_V = 4.73 + 12.86 = \underline{\underline{17.59 \text{ k/ft}}}$$

$$p_{VD} = \gamma_1 h_1$$

$$p_{VD} = 0.12(13) = 1.56 \text{ ksf} = \text{vertical pressure at } D$$

$$p_{HD} = K_1 p_{VD}$$

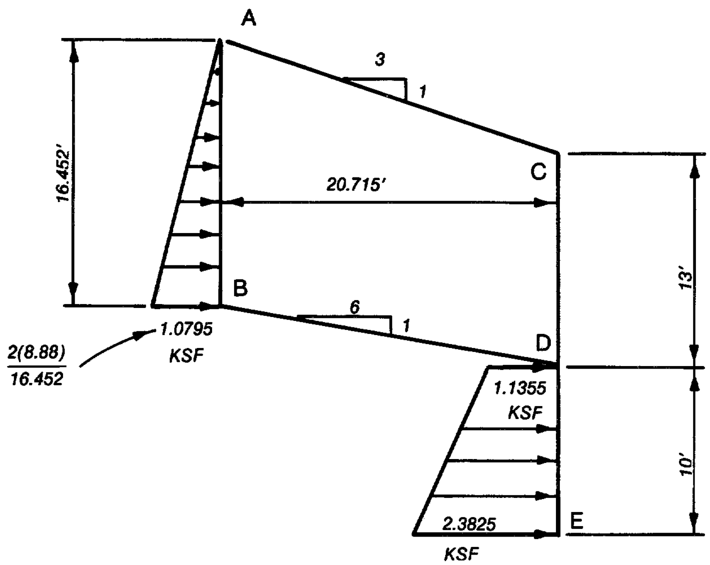
$$p_{HD} = 0.7279 p_{VD} = 1.1355 \text{ ksf} = \text{horizontal pressure at } D$$

$$\frac{1}{2}\,(p_{HD} + p_{HE})\,h_2 = P_{EE2}$$

$$p_{HE} = \frac{2P_{EE2}}{h_2} - p_{HD} = \text{horizontal pressure at } E$$

$$p_{HE} = \frac{2(17.59)}{10} - 1.1355 = 2.3825 \text{ ksf}$$

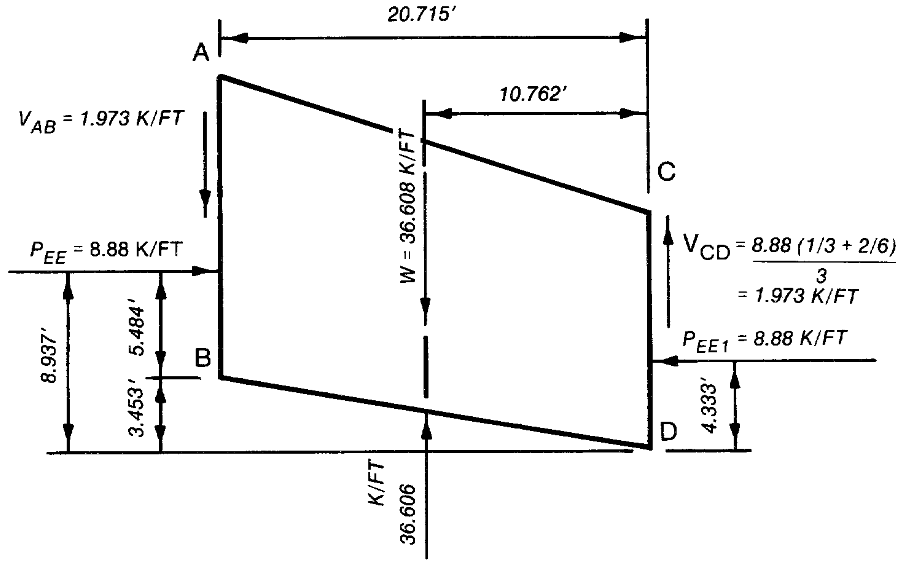
Pressure distributions on Surfaces AB and DE




e. Earth pressure on Surfaces CD and DE. The earth force on AB may be transferred to Surface CD by assuming that no shear resistance is developed on Surface BD. Then to obtain moment equilibrium for Block ABCD, a vertical shear force must be developed on Surfaces AB and CD. The shear force will be:

$$v_{AB} = -v_{CD} = P_{EE1}\left(\frac{\tan \beta_1 + \tan \beta_2}{3}\right)$$

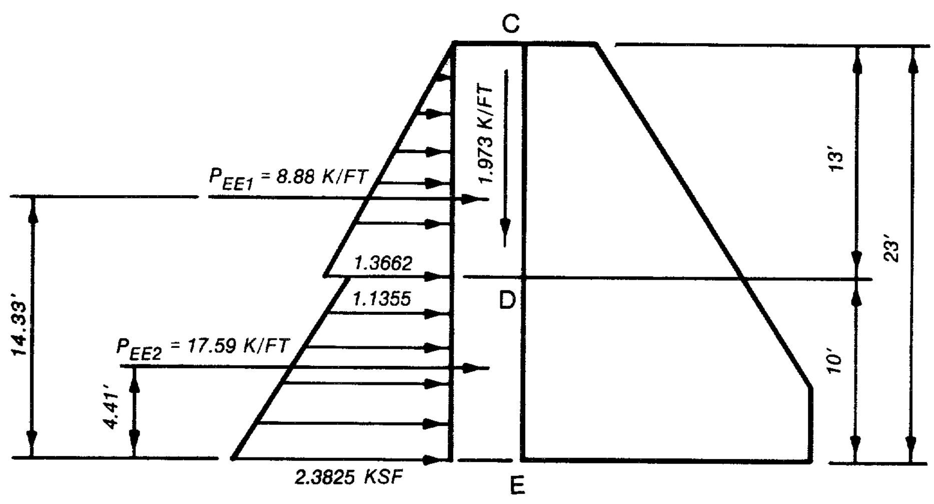
A free body of Block ABCD is shown below.



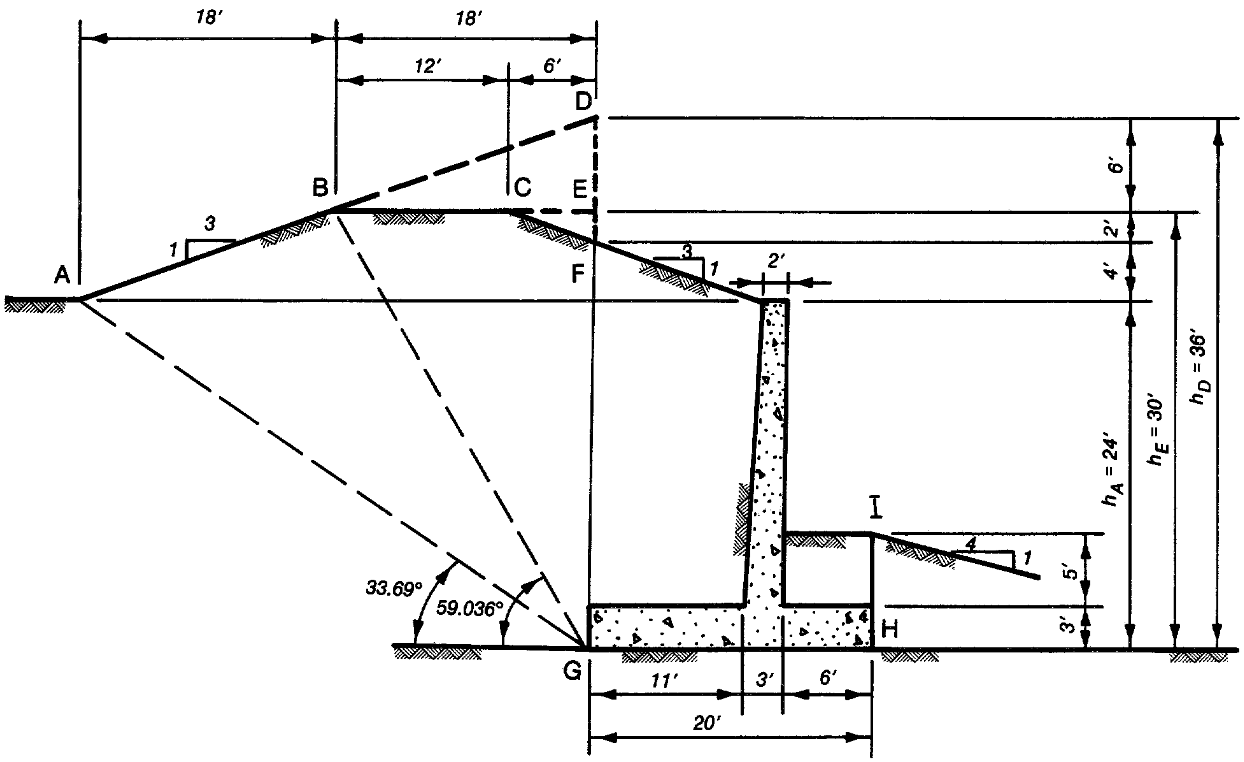

$$\Sigma M_D = 0$$

$$\Sigma M_D = 8.88(8.937 - 4.333) - 1.973(20.715) = 0.013 \approx 0$$

The pressure distribution on Surfaces CD and DE is shown below.



M-7. EXAMPLE 7. Find the lateral earth force and pressure distribution on Surfaces FG and HI when: $\phi = 35°$, $c = 0$, $\gamma = 0.12$ kcf, and SMF = 2/3 .




a. Driving side:

(1) Assume that the critical slip plane intersects Surface BC. $\beta = 0$, $h = h_E = 30$ ft. The weight of the triangular area CEF will be taken as a negative strip surcharge. V = -(1/2)(0.12)(6)(2) = -0.72 k/ft

$$\phi_d = \tan^{-1}\left(\frac{2}{3}\tan 35°\right) = 25°, \ \tan\phi_d = 0.466308$$

From Equation 3-30

$$A = \tan \phi_d - \frac{2V\left(1 + \tan^2 \phi_d\right)}{\gamma h^2}$$

$$A = 0.466308 - \frac{2(-0.72)(1.217443)}{0.12(30)^2} = 0.482541$$

From Equation 3-28

$$c_1 = \frac{2 \tan^2 \phi_d}{A}$$

$$c_1 = \frac{2(0.466308)^2}{0.482541} = 0.901242$$

From Equation 3-29

$$c_2 = \frac{\tan \phi_d}{A}$$

$$c_2 = \frac{0.466308}{0.482541} = 0.966359$$

$$\alpha' = \tan^{-1}\left(\frac{c_1 + \sqrt{c_1^2 + 4c_2}}{2}\right) = 56.866° < 59.036°$$

This shows that the critical slip plane does not intersect Surface BC.

(2) Assume that critical slip plane intersects Surface AB. $\tan \beta = -(1/3)$, $h = h_D = 36$ ft. The weights of areas BDE and CEF will be taken as a negative strip surcharge.

$$V = \frac{1}{2}(0.12)(18)(6) - \frac{1}{2}(0.12)(6)(2) = -7.2 \text{ k/ft}$$

From Equation 3-30

$$A = 0.466308 - \frac{2(-7.2)(1.217443)}{0.12(36)^2} = 0.579034$$

From Equation 3-28

$$c_1 = \frac{2(0.466308)^2 - \dfrac{4(-7.2)\left(-\frac{1}{3}\right)(1.217443)}{0.12(36)^2}}{0.579034} = 0.621268$$

From Equation 3-29

$$c_2 = \frac{0.466308\left[1 - 0.466308\left(-\frac{1}{3}\right)\right] - \left(-\frac{1}{3}\right) + \dfrac{2(-7.2)\left(\frac{1}{9}\right)(1.217443)}{0.12(36)^2}}{0.579034}$$

$$c_2 = 1.484537$$

$$\alpha = \tan^{-1}\left(\frac{c_1 + \sqrt{c_1^2 + 4c_2}}{2}\right) = \underline{\underline{57.473^\circ}}$$

$$33.69^\circ < \alpha < 59.036^\circ$$

This shows that critical slip plane does intersect Surface AB.

(3) Calculate the pressure coefficients (see Appendix H).

$$K = \frac{1 - \tan\phi_d \cot\alpha}{1 + \tan\phi_d \tan\alpha} = \frac{1 - 0.466308(0.637733)}{1 + 0.466308(1.568054)} = 0.405858$$

$$K_1 = K\left(\frac{\tan\alpha}{\tan\alpha - \tan\beta}\right) = 0.405858\left(\frac{1.568054}{1.901387}\right) = 0.334707$$

$$K_v = K \tan \alpha = 0.405858(1.568054) = 0.636407$$

$$P_\gamma = \frac{1}{2} K_1 \gamma h_D^2 = \frac{1}{2} (0.334707)(0.12)(36)^2 = 26.027 \text{ k/ft}$$

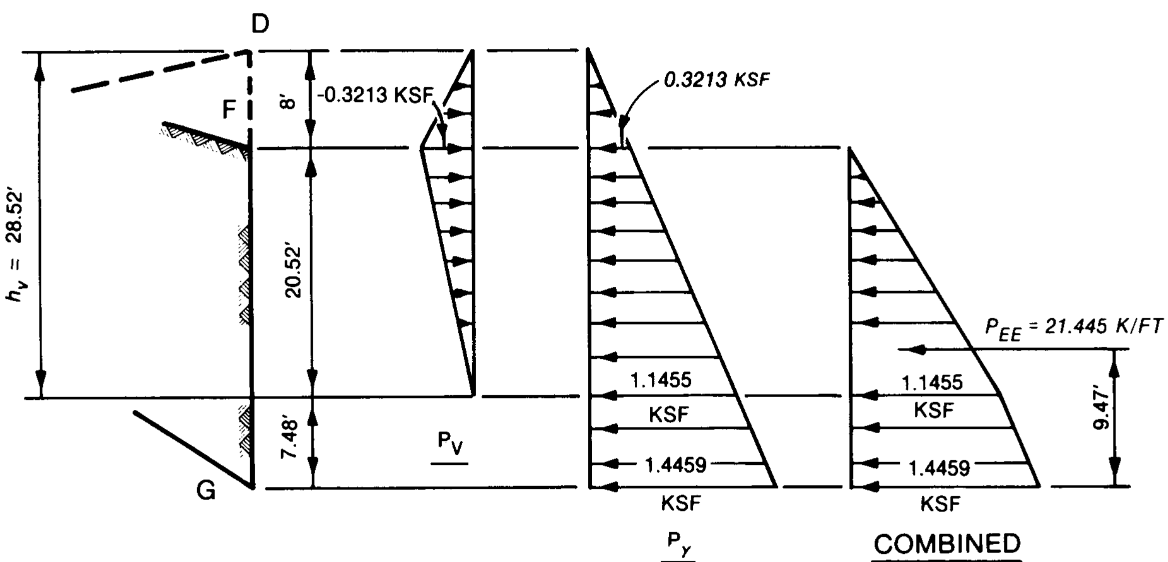
$$P_v = K_v V = 0.636407(-7.2) = -4.582 \text{ k/ft}$$

$$P_{EE} = P_\gamma + P_v = 26.027 - 4.582 = \underline{\underline{21.445 \text{ k/ft}}}$$

(4) The net horizontal pressure at point F must be equal to zero, the negative pressure due to $P_v$ cancels the positive pressure due to $P\gamma$ .

$$p_{\gamma F} = 0.334707(0.12)(8) = 0.3213 \text{ ksf}, \quad p_{vF} = -0.3213 \text{ ksf}$$

$$h_v \text{ (distribution length for } P_v) = \frac{2P_v}{p_{vF}} = \frac{2(-4.582)}{-0.3213} = 28.52 \text{ ft}$$




b. Resisting side:

(1) Calculate the critical slip plane angle $\alpha$ .

$$\tan \beta = -\frac{1}{4}, \quad \tan \phi_d = 0.466308$$

From Equation 3-38

$$A = 0.466308$$

From Equation 3-36

$$c_1 = \frac{2 \tan^2 \phi_d}{A}$$

$$c_1 = \frac{2(0.466308)^2}{0.466308} = 0.932616$$

From Equation 3-37

$$c_2 = \frac{\tan \phi_d \, (1 + \tan \phi_d \tan \beta) + \tan \beta}{A}$$

$$c_2 = \frac{0.466308 \left[1 + 0.466308 \left(- \frac{1}{4}\right)\right] + \left(- \frac{1}{4}\right)}{0.466308} = 0.347297$$

$$\alpha = \tan^{-1}\left(\frac{-c_1 + \sqrt{c_1^2 + 4c_2}}{2}\right) = 15.917° \qquad [3-35]$$

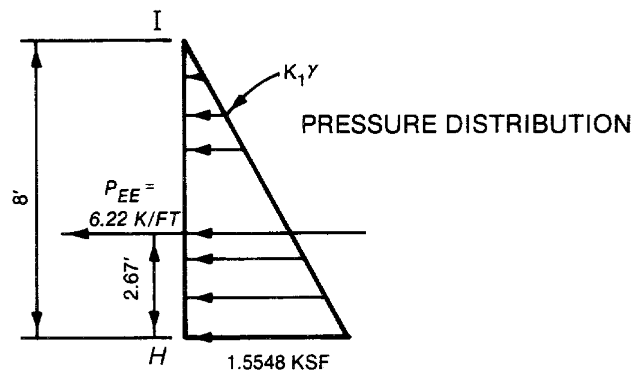
(2) Calculate the pressure coefficients (see Appendix H) and pressure distribution.

$$K = \frac{1 + \tan \phi_d \cot \alpha}{1 - \tan \phi_d \tan \alpha} = \frac{1 + 0.466308(3.506578)}{1 - 0.466308(0.285178)} = 3.039316$$

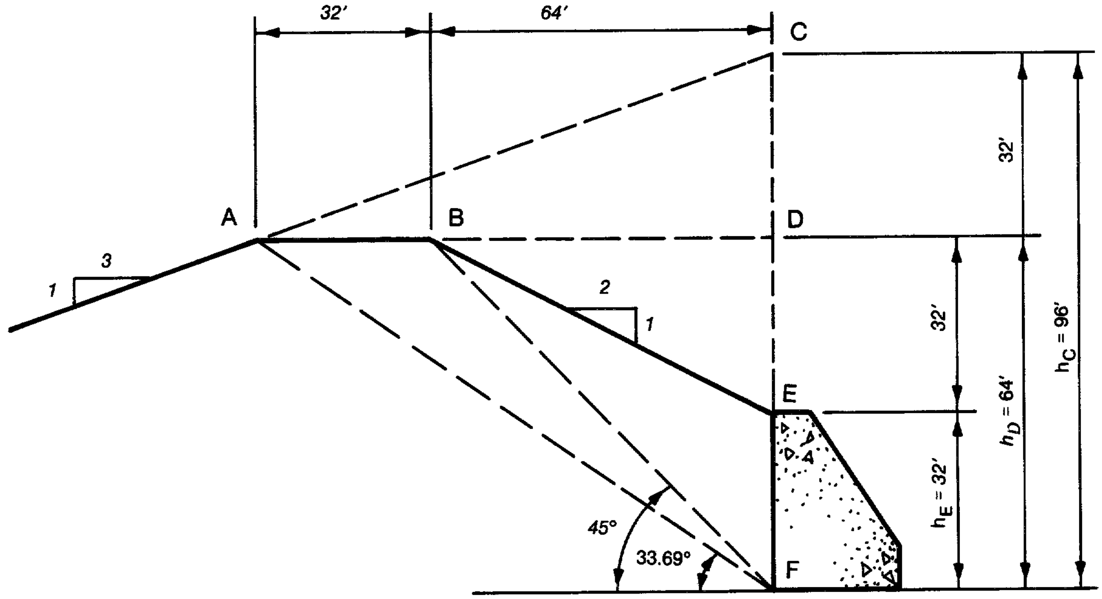
$$K_1 = K \left(\frac{\tan \alpha}{\tan \alpha - \tan \beta}\right) = 3.039316 \left(\frac{0.285178}{0.535178}\right) = 1.619547$$

$$P_{EE} = \frac{1}{2} K_1 \gamma h^2 = \frac{1}{2} (1.619547)(0.12)(8)^2 = \underline{6.22\ k/ft}$$

The calculation of the passive force and pressure distribution as performed above is adequate for performing a sliding analysis but should be calculated as described in paragraph 3-8 when performing an overturning or bearing capacity analysis and for design of structural members.



M-8. EXAMPLE 8. Find the lateral earth force on the wall when:
1. $\phi = 30°$, $c = 0$ , $\gamma = 0.120$ kcf, SMF = 2/3
2. $\phi = 0$ , $c = 0.60$ ksf, $\gamma = 0.120$ kcf, SMF = 2/3




a. $\phi = 30°$, $\phi_d = \tan^{-1}(2/3 \tan \phi) = 21°$, $\tan \phi_d = 0.383864$.

(1) Since the $\tan \beta$ for Surface BE is 0.5 which is greater than $\tan \phi_d$ , the critical slip plane will not intersect BE. Assume that the slip plane intersects Surface AB:

$$\tan \beta = 0 \ , \quad h = h_D = 64 \text{ ft}$$

$$V = -\frac{1}{2}(0.12)[64(32)] = -122.88 \text{ k/ft} \quad \text{(negative weight of area BDE)}$$

From Equation 3-30

$$A = \tan \phi_d - \frac{2V\left(1 + \tan^2 \phi_d\right)}{\gamma h_D^2}$$

$$A = 0.383864 - \frac{2(-122.88)(1.147352)}{0.12(64)^2} = 0.957540$$

From Equation 3-28

$$c_1 = \frac{2 \tan^2 \phi_d}{A}$$

$$c_1 = \frac{2(0.383864)^2}{0.957540} = 0.307771$$

From Equation 3-29

$$c_2 = \frac{\tan \phi_d}{A}$$

$$c_2 = \frac{0.383864}{0.957540} = 0.400886$$

$$\alpha = \tan^{-1}\left(\frac{c_1 + \sqrt{c_1^2 + 4c_2}}{2}\right) = \underline{\underline{38.851°}}$$

$33.69° < \alpha < 45°$, assumption that slip plane intersects Surface AB is correct.

(2) Calculate the pressure coefficients (see Appendix H)

$$K = \frac{1 - \tan \phi_d \cot \alpha}{1 + \tan \phi_d \tan \alpha} = \frac{1 - 0.383864(1.241485)}{1 + 0.383864(0.805487)} = 0.399816$$

$$K_v = K \tan \alpha = 0.399816(0.805487) = 0.322047$$

$$P_\gamma = \frac{1}{2} K\gamma h^2 = \frac{1}{2}(0.399816)(0.12)(64)^2 = 98.26 \text{ k/ft}$$

$P_V = K_V V = 0.322047(-122.88) = -39.57$ k/ft

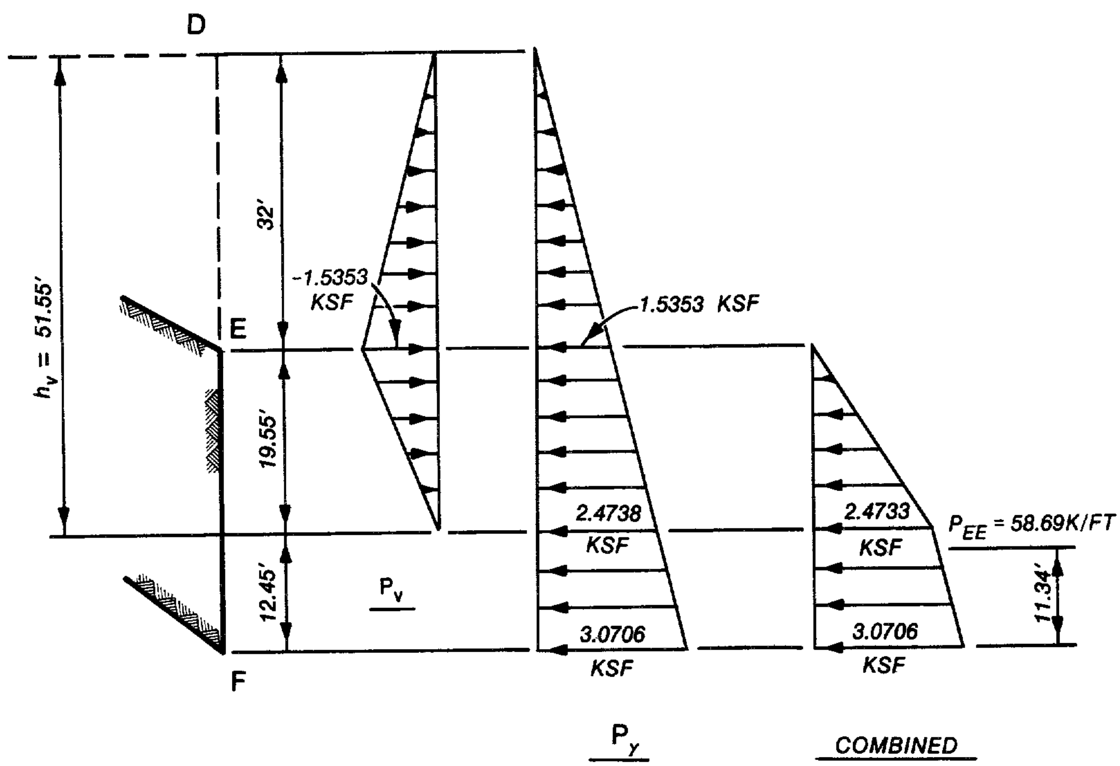
$P_{EE} = P_\gamma + P_V = 98.26 - 39.57 = \underline{\underline{58.69 \text{ k/ft}}}$

(3) The net pressure at point E must be equal to zero. The negative pressure due to $P_V$ cancels the positive pressure due to $P_\gamma$ .

$P_{\gamma E} = 0.399816(0.12)(32) = 1.5353$ ksf

$P_{VE} = -1.5353$ ksf

$$h_v \text{ (distribution length for } P_v) = \frac{2P_v}{P_{vE}} = \frac{2(-39.57)}{-1.5353} = 51.55 \text{ ft}$$



b. $\phi = 0$ , $c = 0.6$ ksf, $c_d = 2/3$ , $c = 0.4$ ksf.

(1) Assume that the vertical crack extends upward from the critical slip plane to intersect Surface BE.

$\tan \beta = 0.5$ , $h = h_E = 32$ ft

From the equations given in Appendix H

$$K_1 = \frac{1 - \tan \phi_d \cot \alpha}{1 + \tan \phi_d \tan \alpha} \cdot \frac{\tan \alpha}{\tan \alpha - \tan \beta} = \frac{\tan \alpha}{\tan \alpha - \tan \beta}$$

$$K_c = \frac{1}{2 \sin \alpha \cos \alpha \, (1 + \tan \phi_d \tan \alpha)} \cdot \frac{\tan \alpha}{\tan \alpha - \tan \beta}$$

$$K_c = \frac{1}{2 \sin \alpha \cos \alpha} \cdot \frac{\tan \alpha}{\tan \alpha - \tan \beta}$$

From Equation I-2

$$d_c = \frac{2K_c c_d}{K_1 \gamma} = \frac{c_d/\gamma}{\sin \alpha \cos \alpha}$$

$$d_c = \frac{0.4/0.12}{\sin \alpha \cos \alpha} = \frac{3.333 \text{ ft}}{\sin \alpha \cos \alpha}$$

Assume that $\alpha = 45°$, then $d_c = \dfrac{3.333}{0.5} = 6.667$ ft

From Equation 3-30

$$A = \frac{2c_d}{\gamma(h + d_c)}$$

$$A = \frac{2(0.4)}{0.12(38.667)} = 0.172412$$

From Equation 3-28

$$c_1 = \frac{\dfrac{4c_d \tan \beta}{\gamma(h + d_c)}}{A}$$

$$c_1 = \frac{\dfrac{4(0.4)(0.5)}{0.12(38.667)}}{0.172412} = 1.00$$

From Equation 3-29

$$c_2 = \frac{-\tan \beta + \dfrac{2c_d}{\gamma(h + d_c)}}{A}$$

$$c_2 = \frac{-0.5 + \dfrac{2(0.4)}{0.12(38.667)}}{0.172412} = -1.900030$$

The term under the radical in Equation 3-25 will be negative. This makes $\alpha$ indeterminate, so the assumption that the crack intersects Surface BE is not correct.

(2) Assume that the vertical crack intersects Surface AB.

$\tan \beta = 0$ , $h = h_d = 64$ ft, $V = -122.88$ k/ft (negative weight of area BDE)

Again assume $\alpha = 45°$, $d_c = 6.667$ ft

From Equations 3-28, 3-29, and 3-30

$$A = \frac{2c_d}{\gamma(h + d_c)} - \frac{2V}{\gamma\left(h^2 - d_c^2\right)}$$

$$A = \frac{2(0.4)}{0.12(70.667)} - \frac{2(-122.88)}{0.12(4051.55)} = 0.599825$$

$$c_1 = 0 \;, \quad c_2 = \frac{2c_d}{\gamma\left(h + d_c\right)} \Bigg/ A = \frac{2(0.4)}{0.12(70.667)} \Bigg/ 0.599825 = 0.157278$$

$$\alpha = \tan^{-1}\left(\frac{c_1 + \sqrt{c_1^2 + 4c_2}}{2}\right) = 21.63° \neq 45° \qquad [3\text{-}25]$$

Let $\alpha = 21.63°$ , $d_c = \dfrac{3.333}{0.342655} = 9.727$ ft

$$A = \frac{2(0.4)}{0.12(73.727)} - \frac{2(-122.88)}{0.12(4001.39)} = 0.602246$$

$$c_1 = 0 \;, \quad c_2 = \frac{2(0.4)}{0.12(73.727)} \Bigg/ 0.602246 = 0.150144$$

$$\alpha = \tan^{-1}\left(\frac{c_1 + \sqrt{c_1^2 + 4c_2}}{2}\right) = 21.18° \neq 21.63°$$

Let $\alpha = 21.18°$ , $d_c = \dfrac{3.333}{0.336893} = 9.893$ ft

$$A = \frac{2(0.4)}{0.12(73.893)} - \frac{2(-122.88)}{0.12(3998.13)} = 0.602460$$

$$c_1 = 0 \;, \quad c_2 = \frac{2(0.4)}{0.12(73.893)} \Bigg/ 0.602460 = 0.149754$$

$\alpha = 21.15° \neq 21.18°$

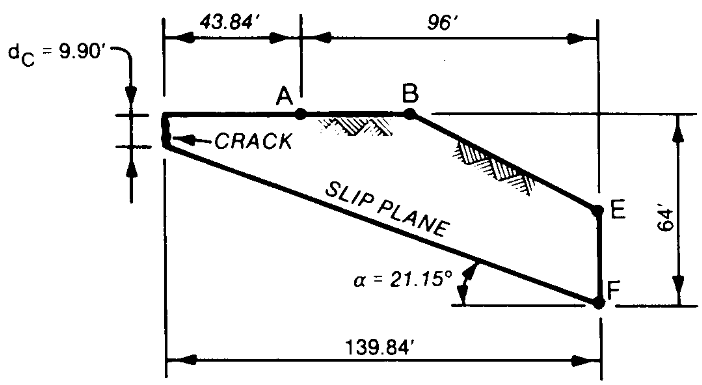
Let $\alpha = 21.15°$ , $d_c = \dfrac{3.333}{0.336506} = 9.90$ ft

$$A = \frac{2(0.4)}{0.12(73.90)} - \frac{2(-122.88)}{0.12(3997.99)} = 0.602469$$

$$c_1 = 0 \ , \quad c_2 = \frac{2(0.4)}{0.12(73.90)} \Big/ 0.602469 = 0.149737$$

$\underline{\underline{\alpha = 21.15°}}$ , $\underline{\underline{d_c = 9.90 \text{ ft}}}$

Check to see if crack intersects Surface AB.




Assumption that crack intersects Surface AB is not correct.

(3) Assume that the crack intersects the sloping surface to the left of point A.

$$\tan \beta = -\frac{1}{3} \ , \quad h = h_c = 96 \text{ ft}, \quad V = -\frac{1}{2} (0.12) \left[(64)(32) + (96)(32)\right]$$

V = -307.20 k/ft (negative weight of areas ACD and BDE)

Let $\alpha = 30°$ , $d_c = \dfrac{3.333}{0.433013} = 7.70$ ft

From Equation 3-30

$$A = \frac{2c_d(1 - \tan\phi \tan\beta)}{\gamma(h + d_c)} - \frac{2V}{\gamma\left(h^2 - d_c^2\right)}$$

$$A = \frac{2(0.4)}{0.12(103.70)} - \frac{2(-307.20)}{0.12(9156.71)} = 0.623441$$

From Equation 3-28

$$c_1 = \frac{\dfrac{4c_d \tan\beta}{\gamma(h + d_c')} - \dfrac{4V \tan\beta}{\gamma\left(h^2 - d_c^2\right)}}{A}$$

$$c_1 = \frac{\dfrac{4(0.4)\left(-\frac{1}{3}\right)}{0.12(103.70)} - \dfrac{4(-307.20)\left(-\frac{1}{3}\right)}{0.12(9156.71)}}{0.623441} = -0.666666$$

From Equation 3-29

$$c_2 = \frac{-\tan\beta + \dfrac{2c_d}{\gamma(h + d_c)} + \dfrac{2V \tan^2\beta}{\gamma\left(h^2 - d_c^2\right)}}{A}$$

$$c_2 = \frac{-\left(-\frac{1}{3}\right) + \dfrac{2(0.4)}{0.12(103.7)} + \dfrac{2(-307.20)\left(\frac{1}{9}\right)}{0.12(9156.71)}}{0.623441} = 0.538132$$

$$\alpha = \tan^{-1}\left(\frac{c_1 + \sqrt{c_1^2 + 4c_2}}{2}\right)$$

$$\alpha = 25.29° \neq 30°$$

Let $\alpha = 25.29°$ , $d_c = \frac{3.333}{0.386256} = 8.63$ ft

$$A = \frac{2(0.4)}{0.12(104.63)} - \frac{2(-307.2)}{0.12(9141.52)} = 0.623799$$

$$c_1 = \frac{\frac{4(0.4)\left(-\frac{1}{3}\right)}{0.12(104.63)} - \frac{4(-307.2)\left(-\frac{1}{3}\right)}{0.12(9141.52)}}{0.623799} = -0.666666$$

$$c_2 = \frac{-\left(-\frac{1}{3}\right) + \frac{2(0.4)}{0.12(104.63)} + \frac{2(-307.2)\left(\frac{1}{9}\right)}{0.12(9141.52)}}{0.623799} = 0.536741$$

$\alpha = 25.25° \neq 25.29°$

Let $\alpha = 25.25°$ , $d_c = \frac{3.333}{0.385812} = 8.64$ ft

$$A = \frac{2(0.4)}{0.12(104.64)} - \frac{2(-307.2)}{0.12(9141.35)} = 0.623803$$

$$c_1 = \frac{\frac{4(0.4)\left(-\frac{1}{3}\right)}{0.12(104.64)} - \frac{4(-307.2)\left(-\frac{1}{3}\right)}{0.12(9141.35)}}{0.623803} = -0.666666$$

$$c_2 = \frac{-\left(-\frac{1}{3}\right) + \frac{2(0.4)}{0.12(104.64)} + \frac{2(-307.2)\left(\frac{1}{9}\right)}{0.12(9141.35)}}{0.623803} = 0.536726$$

$\alpha = \underline{\underline{25.246° \approx 25.25°}}$ , $\underline{\underline{d_c = 8.64 \text{ ft}}}$

(4) Calculate pressure coefficients (see Appendix H) and earth forces:

$$K = \frac{1 - \tan \phi_d \cot \alpha}{1 + \tan \phi_d \tan \alpha} = \frac{1 - 0(2.120303)}{1 + 0(0.471631)} = 1$$

$$K_1 = K \left(\frac{\tan \alpha}{\tan \alpha - \tan \beta}\right) = 1 \left(\frac{0.471631}{0.804964}\right) = 0.585903$$

$$K_v = K \tan \alpha = 1(0.471631) = 0.471631$$

$$P_\gamma = \frac{1}{2} K_1 \gamma \left(h - d_c\right)^2 = \frac{1}{2} (0.585903)(0.12)(96 - 8.64)^2 = 268.29 \text{ k/ft}$$

$$P_v = K_v V = 0.471631(-307.2) = - 144.89 \text{ k/ft}$$

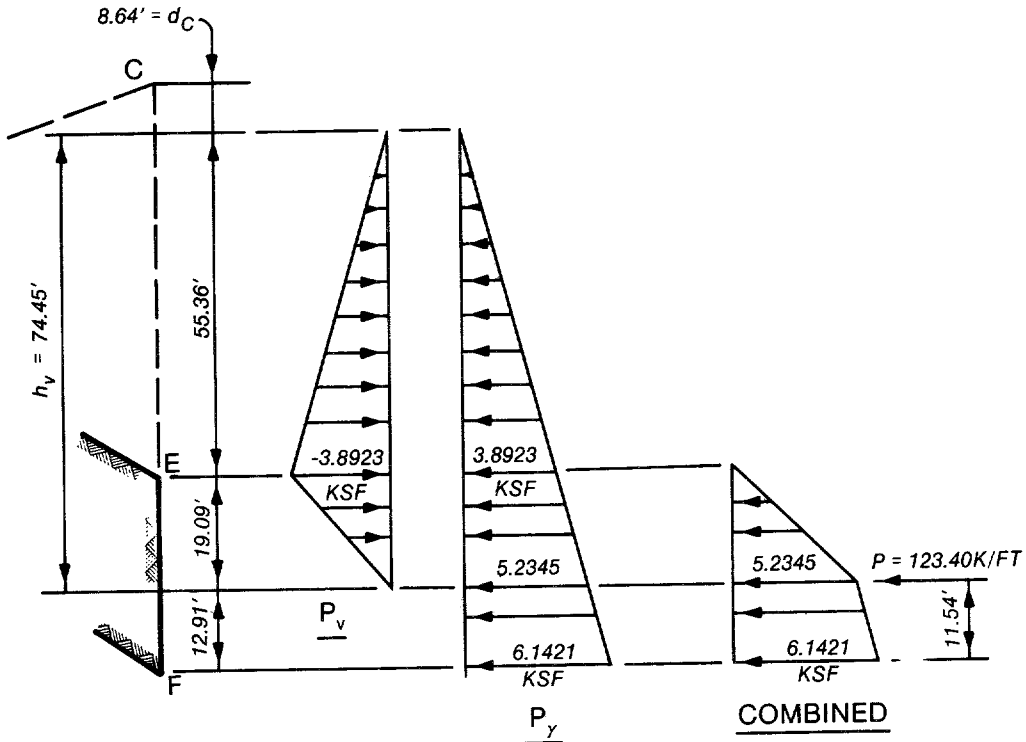
$$P_{EE} = P_\gamma + P_v = 268.29 - 144.89 = \underline{\underline{123.40 \text{ k/ft}}}$$

(5) Pressure distribution. The negative horizontal pressure, due to $P_v$ , at point E must cancel the positive pressure, due to $P_\gamma$ .

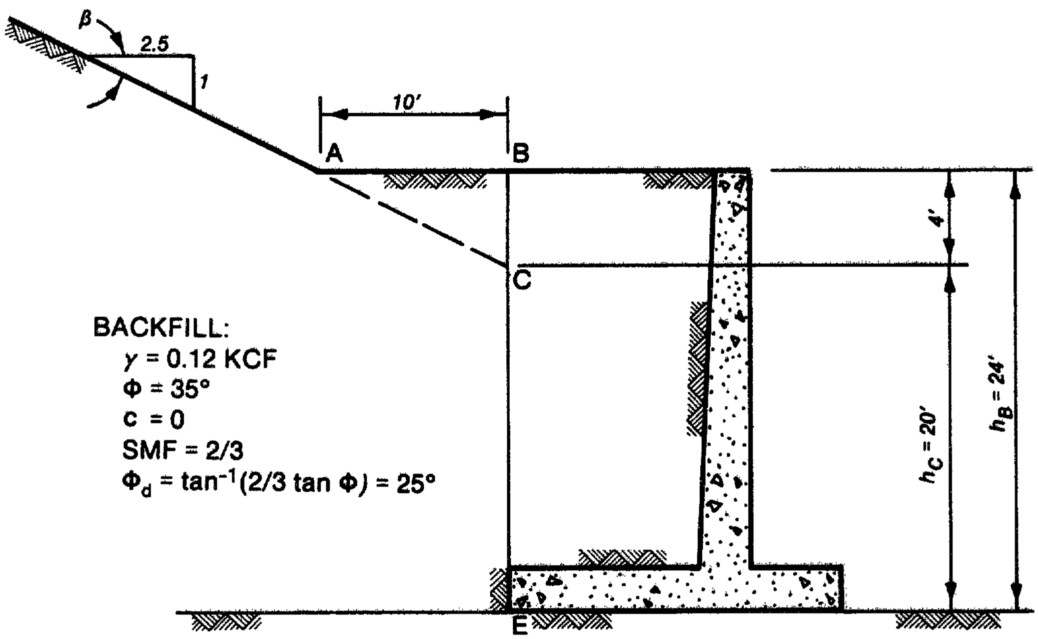
$$p_{\gamma E} = 0.585903(0.12)(64 - 8.64) = 3.8923 \text{ ksf}$$

$$p_{vE} = -3.8923 \text{ ksf}$$

$$h_v = \text{distribution length for } P_v = \frac{2(-144.89)}{-3.8923} = 74.45 \text{ ft}$$


8.64' = $d_C$
C
$h_v$ = 74.45'
55.36'
E
19.09'
12.91'
F
-3.8923
KSF
$P_v$
3.8923
KSF
5.2345
6.1421
KSF
$P_Y$
5.2345
P = 123.40K/FT
11.54'
6.1421
KSF
COMBINED

M-9. EXAMPLE 9. Compute lateral earth pressure using pressure coefficients. Check by wedge method.




a. Find critical slip-plane angle. Consider basic wedge to have height of 20 feet ($h_c$), with sloping top surface (tan β = 0.4). The weight of triangle ABC will be considered a finite surcharge (V).

$$V = \frac{1}{2} (0.12)\ (10)\ (4) = 2.4\ k$$

From Equation 3-30

$$A = \tan \phi_d - \frac{2V\left(1 + \tan^2\right)\phi_d}{\gamma h_c^2} = 0.466308 - \frac{2(2.4)(1.217443)}{0.12(20)^2}$$

$$A = 0.344564$$

From Equation 3-28

$$c_1 = \left[2 \tan^2 \phi_d - \frac{4V \tan \beta \left(1 + \tan^2 \phi_d\right)}{\gamma h_c^2}\right] \div A$$

$$c_1 = \left[2(0.217443) - \frac{4(2.4)(0.4)(1.217443)}{0.12(20)^2}\right] \div 0.344564$$

$$c_1 = 0.979471$$

From Equation 3-29

$$c_2 = \left[\tan \phi_d \ (1 - \tan \phi_d \tan \beta) - \tan \beta + \frac{2V \tan^2 \beta \left(1 + \tan \phi_d\right)^2}{\gamma h_c^2}\right] \div A$$

$$c_2 = \left[0.466308(1 - 0.466308 \times 0.4) - 0.4 + \frac{2(2.4)(0.16)(1.217443)}{0.12(20)^2}\right]$$

$$\div 0.344564$$

$$c_2 = -0.003454$$

$$\alpha = \tan^{-1}\left(\frac{c_1 + \sqrt{c_1^2 + 4c_2}}{2}\right) = \underline{\underline{44.302°}} \qquad [3\text{-}25]$$

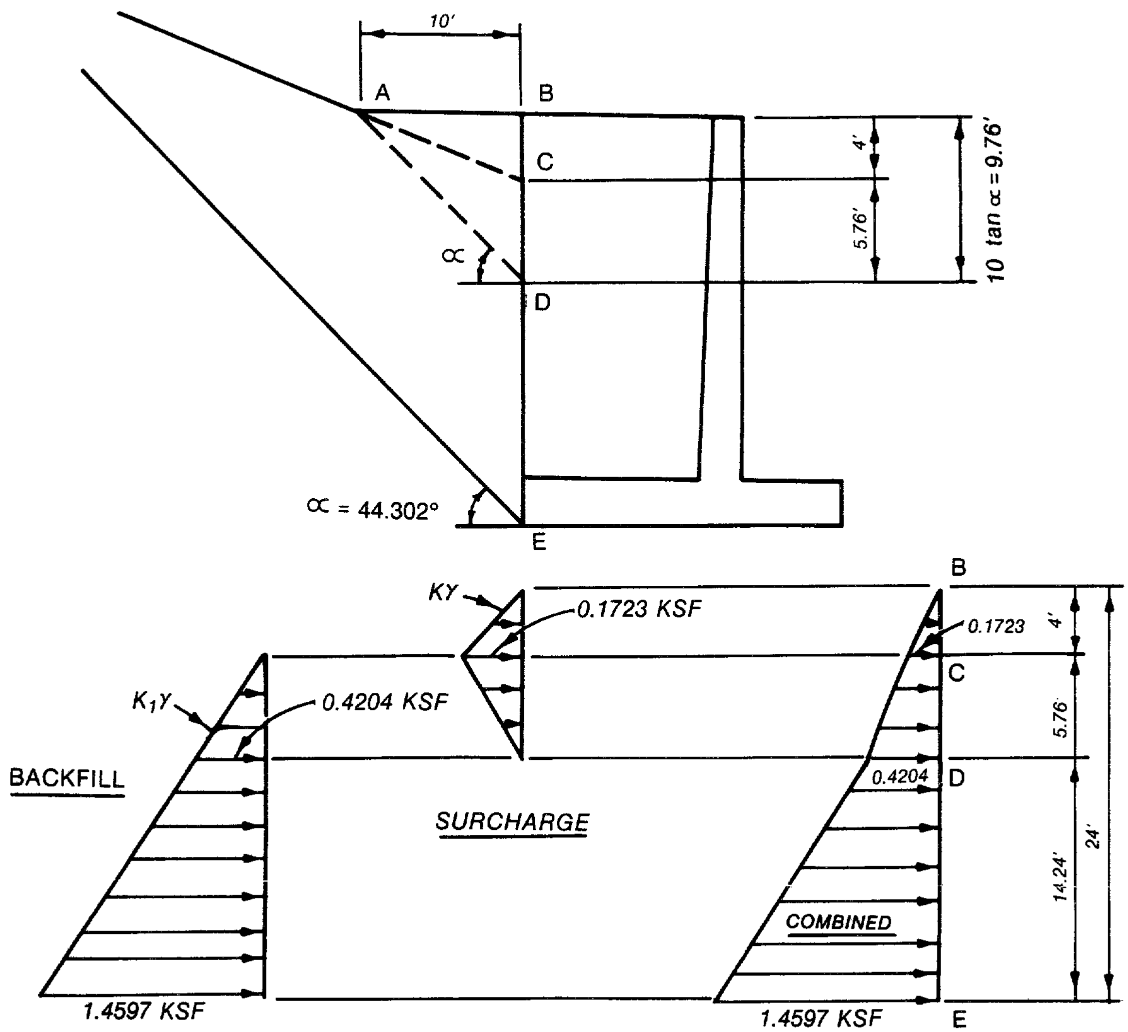
b. <u>Calculate earth pressure coefficients (Appendix H)</u>.

$$K = \frac{1 - \tan \phi_d \cot \alpha}{1 + \tan \phi_d \tan \alpha} = \frac{1 - 0.466308 \times 1.024667}{1 + 0.466308 \times 0.975927}$$

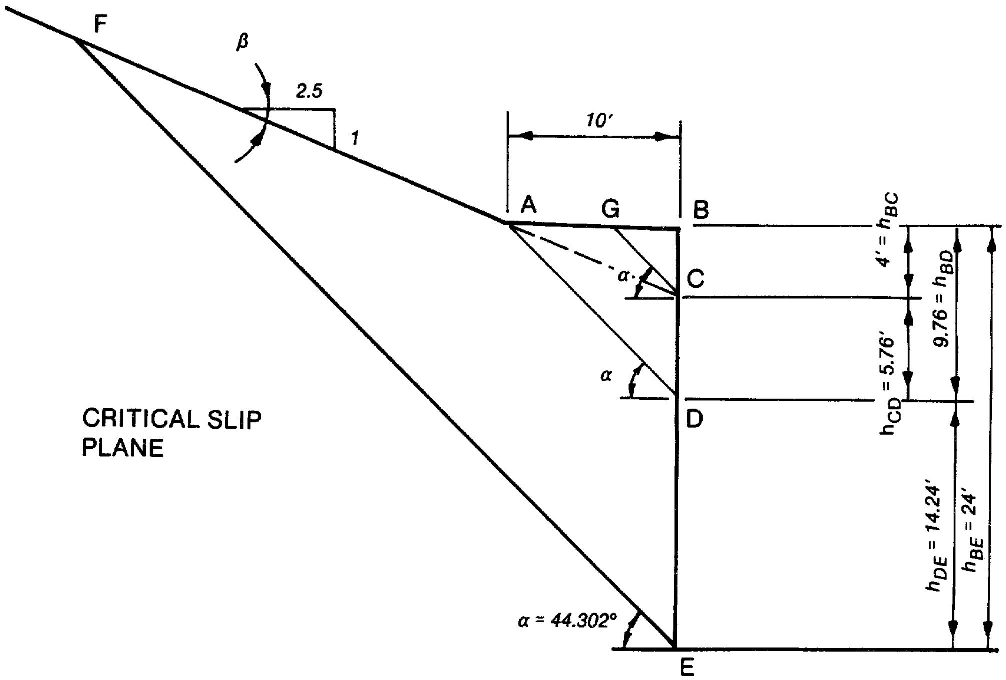
$$K = \underline{\underline{0.3589}}$$

$$K_1 = K\left(\frac{\tan\alpha}{\tan\alpha - \tan\beta}\right) = 0.3589\left(\frac{0.975927}{0.575927}\right) = \underline{\underline{0.6082}}$$

The earth pressure distribution is calculated in the following figures.



c. Check pressure using the wedge method.




Force on Surface BC.

$$\text{Weight of wedge GBC} = W = \frac{\gamma h_{BC}^2}{2 \tan \alpha} = \frac{0.12(4)^2}{2(0.975927)} = 0.9837 \text{ k}$$

From Equation 3-23

$$P_{BC} = \frac{W (\tan \alpha - \tan \phi_d)}{1 + \tan \phi_d \tan \alpha} = \frac{0.9837(0.975927 - 0.466308)}{1 + 0.466308(0.975927)} = \underline{\underline{0.345 \text{ k}}}$$

Force on Surface BD.

$$\text{Weight of wedge ABD} = W = \frac{1}{2} (0.12)(10)(9.76) = 5.8560 \text{ k}$$

$$P_{BD} = \frac{5.8560(0.975927 - 0.466308)}{1 + 0.466308(0.975927)} = \underline{\underline{2.051\ k}}$$

Force on Surface BE.

$$\text{Weight of wedge FABE} = W = \frac{\gamma h_{CE}^2}{2\ (\tan\alpha - \tan\beta)} + \frac{1}{2}\ (0.12)(10)(4)$$

$$W = \frac{0.12(20)^2}{2(0.975927 - 0.4)} + 2.4 = 44.0719\ k$$

$$P_{BE} = \frac{44.0719(0.975927 - 0.466308)}{1 + 0.466308(0.975927)} = \underline{\underline{15.435\ k}}$$

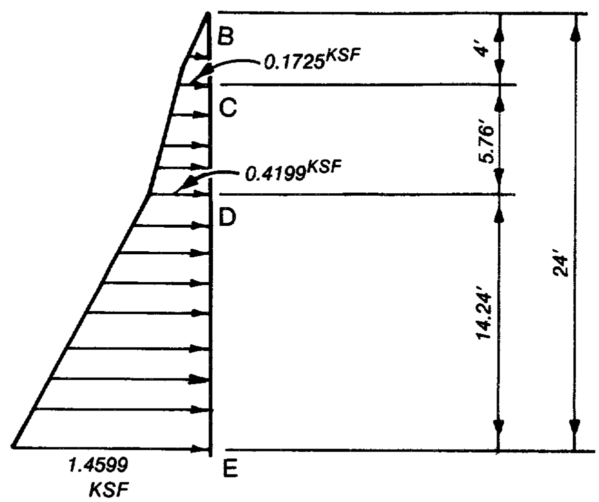
Calculate pressures at C, D, and E.

$$p_C = \frac{2P_{BC}}{h_{BC}} = \frac{2(0.345)}{4} = 0.1725\ \text{ksf}$$

$$p_D = \frac{2(P_{BD} - P_{BC})}{h_{CD}} - p_C = \frac{2(2.051 - 0.345)}{5.76} - 0.1725 = 0.4199\ \text{ksf}$$

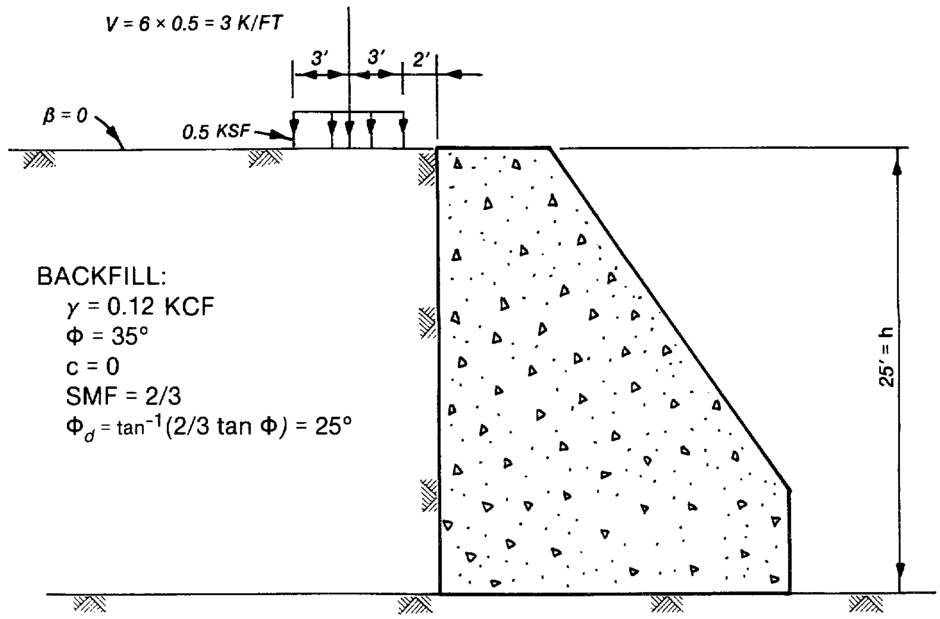
$$p_E = \frac{2(P_{BE} - P_{BD})}{h_{DE}} - p_D = \frac{2(15.435 - 2.051)}{14.24} - 0.4199 = 1.4599\ \text{ksf}$$

The pressure diagram-wedge method is shown on the following page.




Checks combined pressure diagram obtained using pressure coefficients.

M-10. EXAMPLE 10. Calculate lateral earth pressure. Use the approximate wedge method as well as the elastic method to find pressure due to the distributed finite surcharge.




a. Find critical slip-plane angle and lateral force using pressure coefficients. From Equation 3-30

$$A = \tan \phi_d - \frac{2V\ (1 + \tan^2\ \phi_d)}{\gamma h^2} = 0.466308 - \frac{2(3)(1.217443)}{0.12(25)^2}$$

$$A = 0.368913$$

From Equation 3-28

$$c_1 = \frac{2\ \tan^2\ \phi_d}{A} = \frac{2(0.466308)^2}{0.368913} = 1.178832$$

From Equation 3-29

$$c_2 = \frac{\tan \phi_d}{A} = \frac{0.466308}{0.368913} = 1.264005$$

$$\alpha = \tan^{-1}\left(\frac{c_1 + \sqrt{c_1^2 + 4c_2}}{2}\right) = \underline{\underline{61.721°}} \qquad [3\text{-}25]$$

From Appendix H, the pressure coefficients are:

$$K = \frac{1 - \tan \phi_d \cot \alpha}{1 + \tan \phi_d \tan \alpha} = \frac{1 - 0.466308 \times 0.537972}{1 + 0.466308 \times 1.858833}$$

$$K = \underline{\underline{0.4013}}$$

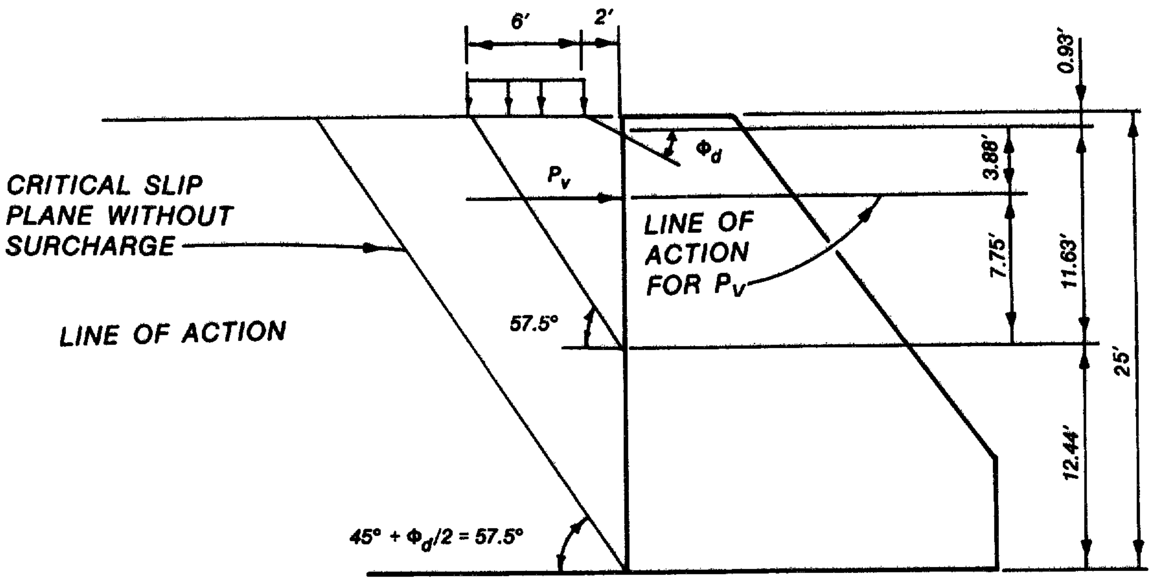
$$K_v = K \tan \alpha = 0.4013(1.858833) = 0.7459$$

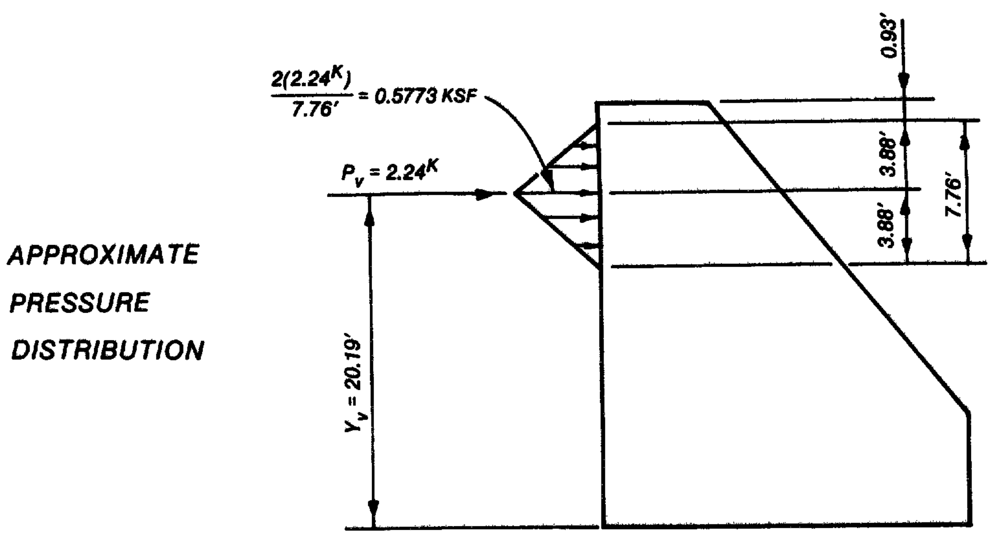
$$P_\gamma = \frac{1}{2} K\gamma h^2 = \frac{1}{2} (0.4013)(0.12)(25)^2 = 15.05 \text{ k}$$

$$P_v = K_v V = 0.7459(3) = 2.24 \text{ k}$$

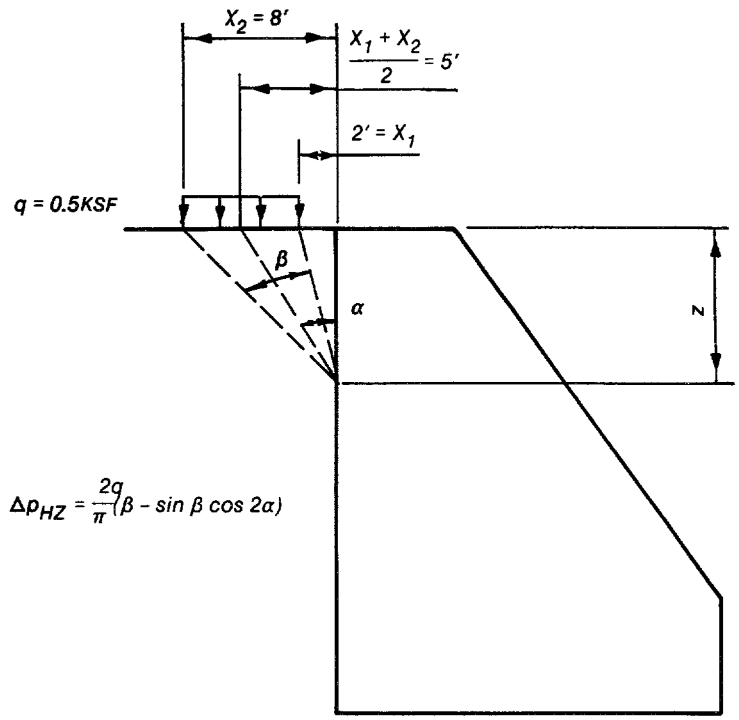
$$P_{(total)} = P_\gamma + P_v = 15.05 + 2.24 = \underline{\underline{17.29 \text{ k}}}$$

Approximate method-pressure distribution for surcharge (see Figure 3-29).



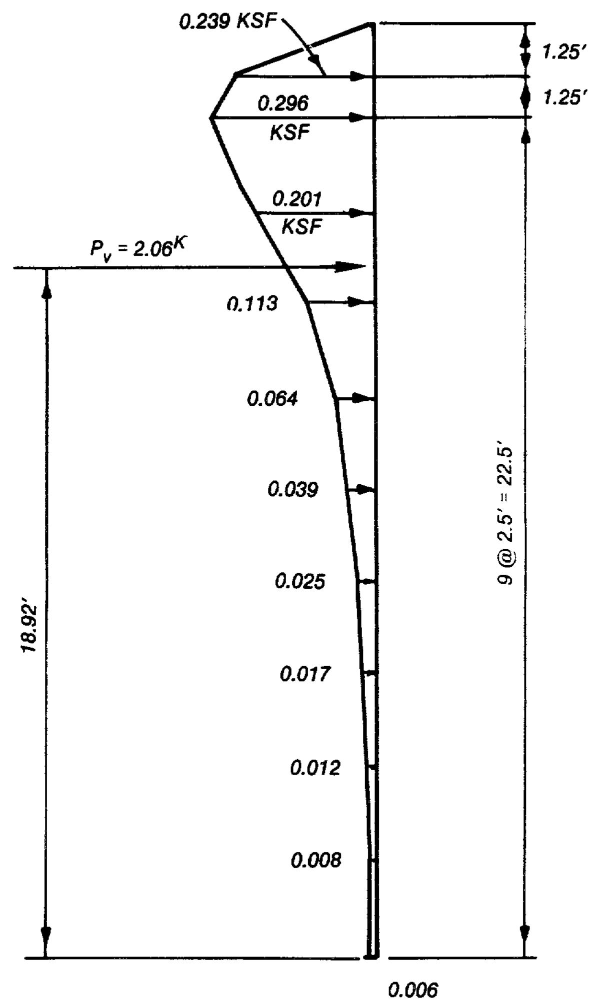




b. <u>Find pressure distribution due to surcharge using the elastic method (see Figure 3-27)</u>. Assume nonyielding wall.



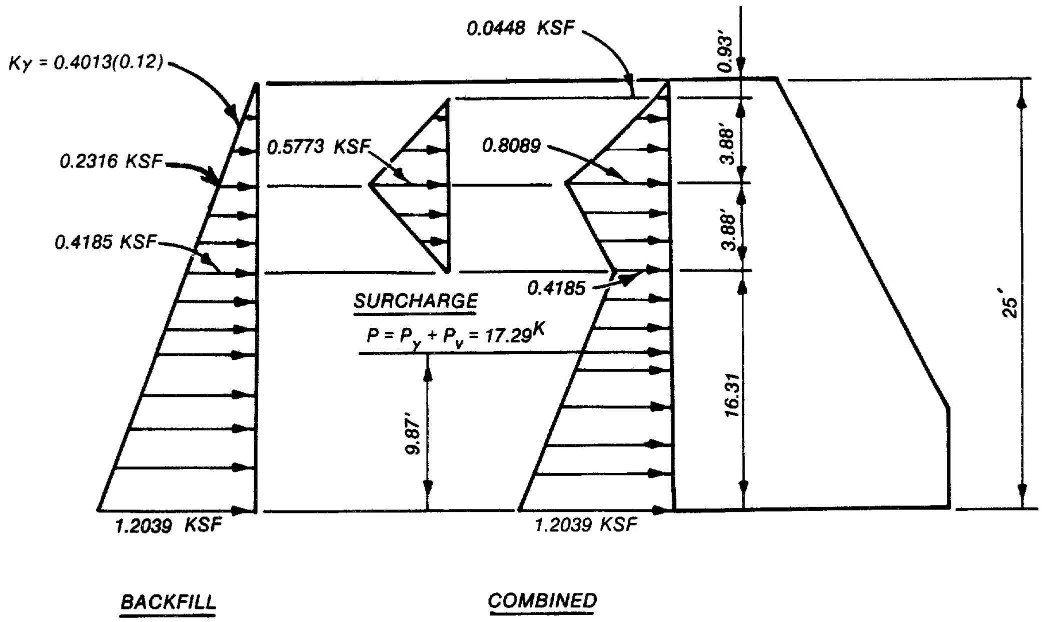

| z , ft | $\frac{2q}{\pi}$ | β , rad | α° | $\Delta p_{Hz}$, ksf |
|---|---|---|---|---|
| 1.25 | 0.3183 | 0.4036 | 75.96 | 0.239 |
| 2.50 | 0.3183 | 0.5932 | 63.43° | 0.296 |
| 5.00 | 0.3183 | 0.6316 | 45.00 | 0.201 |
| 7.50 | 0.3183 | 0.5570 | 36.69 | 0.113 |
| 10.00 | 0.3183 | 0.4773 | 26.57 | 0.064 |
| 12.50 | 0.3183 | 0.4107 | 21.80 | 0.039 |
| 15.00 | 0.3183 | 0.3574 | 18.43 | 0.025 |
| 17.50 | 0.3183 | 0.3150 | 15.95 | 0.017 |
| 20.00 | 0.3183 | 0.2808 | 14.04 | 0.012 |
| 22.50 | 0.3183 | 0.2530 | 12.53 | 0.008 |
| 25.00 | 0.3183 | 0.2298 | 11.31 | 0.006 |

Pressure diagram-elastic method.




The force, due to the surcharge, determined by the approximate method is more severe. It will be combined with the backfill force to obtain the total force.

Combined pressure diagram and force:



M-11. EXAMPLE 11. Find the lateral forces and pressures acting on the wall for the seismic condition.

Soil properties (on both sides of wall):

$\gamma$ = 0.12 k/ft$^3$ (moist weight)

$\gamma_b$ = 0.0625 k/ft$^3$ (buoyancy weight)

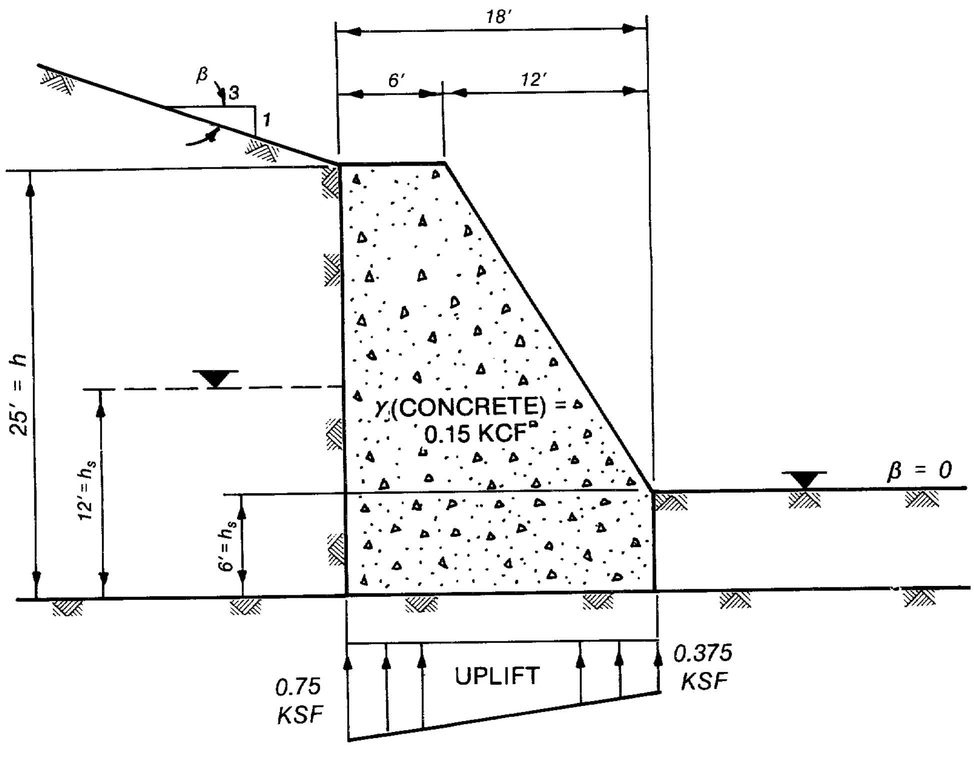
$\gamma_s$ = 0.125 k/ft$^3$ (saturated weight)

$\phi$ = 35° , c = 0

Seismic coefficients:

$k_H$ = 0.20

$k_V$ = 0



a. <u>Find forces acting on driving side</u>.

$$c_1 = \frac{2\ (\tan \phi - k_h)}{1 + k_h \tan \phi} = \frac{2(0.700208 - 0.2)}{1 + 0.2(0.700208)} = 0.877526 \qquad [3\text{-}57]$$

$$c_2 = \frac{\tan \phi\ (1 - \tan \phi \tan \beta) - (\tan \beta + k_h)}{\tan \phi\ (1 + k_h \tan \phi)} \qquad [3\text{-}58]$$

$$c_2 = \frac{0.700208 \left(1 - 0.700208 \times \frac{1}{3}\right) - \left(\frac{1}{3} + 0.2\right)}{0.700208(1 + 0.2 \times 0.700208)} = 0.004315$$

$$\alpha = \tan^{-1} \left( \frac{c_1 + \sqrt{c_1^2 + 4c_2}}{2} \right) = \underline{\underline{41.426°}} \qquad [3\text{-}56]$$

$$K = \frac{1 - \tan \phi \cot \alpha}{1 + \tan \phi \tan \alpha} = \frac{1 - 0.700208(1.133240)}{1 + 0.700208(0.882425)}$$

$$K = 0.12763$$

$$K_A = K \left( \frac{\tan \alpha}{\tan \alpha - \tan \beta} \right) = 0.12763 \left( \frac{0.882425}{0.882425 - \frac{1}{3}} \right) = \underline{\underline{0.2051}} \qquad [3\text{-}54]$$

$$K_b = K \left[ 1 + \left( \frac{\tan \alpha}{\tan \alpha - \tan \beta} - 1 \right) \left( \frac{\gamma}{\gamma_b} \right) \right] \qquad \text{(see Appendix H)}$$

$$K_b = 0.12763 \left[ 1 + \left( \frac{0.882425}{0.549092} - 1 \right) \left( \frac{0.12}{0.0625} \right) \right] = \underline{\underline{0.2764}}$$

$$P_A = \frac{1}{2} K_A \gamma \left(h - h_s\right)^2 + \frac{1}{2} (h_s) \left[ 2K\gamma(h - h_s) + K_b \gamma_b h_s \right] \qquad [3\text{-}69]$$

$$P_A = \frac{1}{2}(0.2051)(0.12)(13)^2 + \frac{1}{2}(12)\left[2(0.2051)(0.12)(13) + 0.2764(0.0625)(12)\right]$$

$$P_A = \underline{\underline{7.16\ k}}$$

$$\Delta P_{AE} = k_h\left[\frac{\gamma h^2}{2\ (\tan\alpha - \tan\beta)} + \frac{(\gamma_s - \gamma)h_s^2}{2\tan\alpha}\right] \qquad [3\text{-}71]$$

$$\Delta P_{AE} = 0.2\left[\frac{0.12(25)^2}{2(0.549092)} + \frac{0.005(12)^2}{2(0.882425)}\right] = \underline{\underline{13.74\ k}}$$

$$P_{ws} = \frac{1}{2}\gamma_w h_s^2 = \frac{1}{2}(0.0625)(12)^2 = \underline{\underline{4.50\ k}} \qquad [3\text{-}70]$$

b. <u>Find forces acting on resisting side</u>.

$$c_1 = \frac{2\ (\tan\phi - k_h)}{1 + k_h\tan\phi} = \frac{2(0.700208 - 0.2)}{1 + 0.2(0.700208)} = 0.877526 \qquad [3\text{-}60]$$

From Equation 3-61

$$c_2 = \frac{\tan\phi - k_h}{\tan\phi\ (1 + k_h\tan\phi)}$$

$$c_2 = \frac{0.700208 - 0.2}{0.700208(1 + 0.2 \times 0.700208)} = 0.626618$$

$$\alpha = \tan^{-1}\left(\frac{-c_1 + \sqrt{c_1^2 + 4c_2}}{2}\right) = \underline{\underline{24.999°}} \qquad [3\text{-}59]$$

From Equation 3-77

$$K_P = \frac{1 + \tan\phi \cot\alpha}{1 - \tan\phi \tan\alpha} = \frac{1 + 0.700208(2.144605)}{1 - 0.700208(0.466286)}$$

$$K_P = \underline{\underline{3.7144}}$$

From Equation 3-73

$$P_P = \frac{1}{2} K_P \gamma_b h^2 = \frac{1}{2}(3.7144)(0.0625)(6)^2 = \underline{\underline{4.18\ k}}$$

From Equation 3-75

$$\Delta P_{PE} = k_h \left(\frac{\gamma_s h^2}{2\tan\alpha}\right) = 0.2\left[\frac{0.125(6)^2}{2(0.466286)}\right] = \underline{\underline{0.97\ k}}$$

$$P_{ws} = \frac{1}{2}\gamma_w h_s^2 = \frac{1}{2}(0.0625)(6)^2 = \underline{\underline{1.13\ k}} \qquad [3\text{-}74]$$

c. <u>Find inertia force due to weight of wall.</u>

$$18' \times 25' \times 0.15 = 67.50 \times 12.50' = 843.75$$

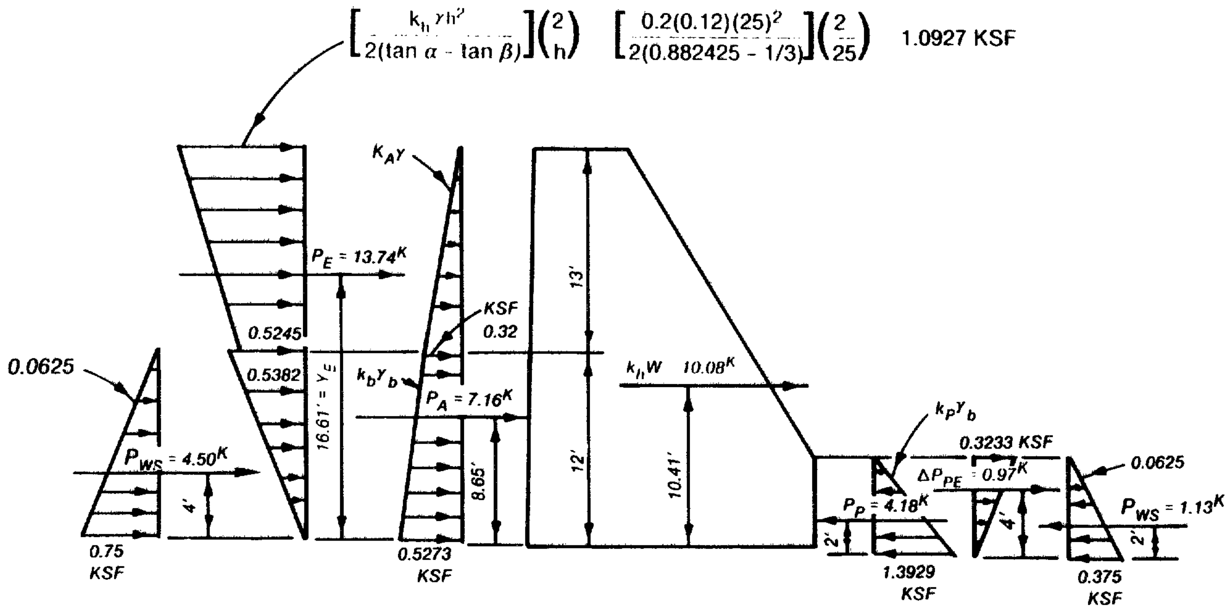
$$-\frac{1}{2} \times 12' \times 19' \times 0.15 = \underline{-17.10} \times 18.67' = \underline{-319.25}$$

$$W = 50.40\ k \qquad 524.50$$

$$\bar{y} = \frac{524.50}{50.40} = \underline{\underline{10.41\ ft}}$$

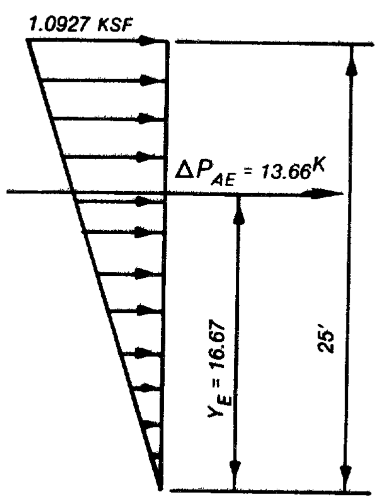
$$k_h W = 0.2(50.40\ k) = \underline{\underline{10.08\ k}}$$

d. Summary of forces and pressure distributions.




Permissible simplification for dynamic earth pressure distribution--driving side:

The discontinuity of this pressure diagram, at the water table, may be eliminated by considering that the soil weight above and below water is equal to the moist weight. The difference is not significant.




In this case, the difference in forces is -0.58% and difference in dimension, $Y_E$ , is +0.36%.

Mononobe-okabe force and pressure distribution--resisting side.

If the pressure diagrams for $P_P$ and $\Delta P_{PE}$ (on the preceding page) are combined, negative pressure will be obtained for some distance below the top of ground. Since earth pressure can not pull on the wall, the pressure diagram and force should be determined by setting all negative pressures to zero.