Appendix B
Mathematical Analysis of Underseepage and Substratum Pressure

B-1. General

The design of seepage control measures for levees often requires an underseepage analysis without the use of piezometric data and seepage measurements. Contained within this appendix are equations by which an estimate of seepage flow and substratum pressures can be made, provided soil conditions at the site are reasonably well defined. The equations contained herein were developed during a study (reported in U.S. Army Engineer Waterways Experiment Station TM 3-424 (Appendix A) of piezometric data and seepage measurements along the Lower Mississippi River and confirmed by model studies. It should be emphasized that the accuracy obtained from the use of equations is dependent upon the applicability of the equation to the condition being analyzed, the uniformity of soil conditions, and evaluation of the various factors involved. As is normally the case, sound engineering judgment must be exercised in determining soil profiles and soil input parameters for these analyses.

B-2. Assumptions

It is necessary to make certain simplifying assumptions before making any theoretical seepage analysis. The following is a list of such assumptions and criteria necessary to the analysis set forth in this appendix.

a. Seepage may enter the pervious substratum at any point in the foreshore (usually at riverside borrow pits) and/or through the riverside top stratum.

b. Flow through the top stratum is vertical.

c. Flow through the pervious substratum is horizontal.

d. The levee (including impervious or thick berms) and the portion of the top stratum beneath it is impervious.

e. All seepage is laminar.

In addition to the above, it is also required that the foundation be generalized into a pervious sand or gravel stratum with a uniform thickness and permeability and a semipervious or impervious top stratum with a uniform thickness and permeability (although the thickness and permeability of the riverside and landside top stratum may be different).

B-3. Factors Involved in Seepage Analyses

The volume of seepage (Q_s) that will pass beneath a levee and the artesian pressure that can develop under and landward of a levee during a sustained high water are related to the basic factors given and defined in Table B-1 and shown graphically in Figure B-1. Other values used in the analyses are defined as they are discussed in subsequent paragraphs.

B-4. Determination of Factors Involved in Seepage Analyses

Table B-2 contains a brief summary of methods normally used to determine the factors necessary to perform a seepage analysis. The determination of these factors is discussed in more detail in the following paragraphs. Many of the methods given, such as exploration and testing, have previously been mentioned in the text; however, they will be discussed herein in more detail as they apply to each specific factor. The use of piezometric data, although rarely available on new projects, is mentioned primarily because it is not infrequent for seepage analyses to be performed as a part of remedial measures to existing levees in which case piezometric data often are available.

Table B-1
Factors Involved in Seepage Analyses

Factor	Definition
H	Net head on levee
M	Slope of hydraulic grade line (at middepth of pervious stratum) beneath levee
i_c	Critical gradient for landside top stratum
L_1	Distance from river to riverside levee toe
L_2	Base width of levee and berm
L_3	Length of foundation and top stratum beyond landside levee toe
L	Distance from effective seepage entry to effective seepage exit
s	Distance from effective seepage entry to landside toe of levee or berm
X_1	Distance from effective seepage entry to riverside levee toe
X_3	Distance from landside levee toe to effective seepage exit
d	Thickness of pervious substratum
z	Thickness of top stratum
z_b	Transformed thickness of top stratum
z_{bl}	Transformed thickness of landside top stratum
z_{br}	Transformed thickness of riverside top stratum
z_n	Thickness of individual layers comprising top stratum (n = layer number)
z_t	Transformed thickness of landside top stratum for uplift computation
k_b	Vertical permeability of top stratum
k_{bl}	Vertical permeability of landside top stratum
k_{br}	Vertical permeability of riverside top stratum
k_f	Horizontal permeability of pervious substratum
k_n	Vertical permeability of individual layers comprising top stratum (n = layer number)
Q_s	Total amount of seepage passing beneath the levee
h_o	Head beneath top stratum at landside levee toe
h_x	Head beneath top stratum at distance x from landside levee toe

Table B-2
Methods for Determination of Design Parameters

Factor	Method of Determination
H	From design flood stage or net levee grade
k_{bl}, k_{br}	From laboratory tests, estimations, and transformations
k_f	Field pump tests, correlations
z_b	Foundation exploration, knowledge of depth and locations of borrow pits, ditches, etc.
z_{bl} , z_{br}	From transformations
d	Foundation exploration
i_c	From equation B-9
M	From piezometers or from determining effective entrance and exit points of seepage
L_1	From maps
L_2	From preliminary or existing levee section
L_3	From foundation exploration and knowledge of location of levee
s	From piezometric data or estimated from equations
x_1	From knowing M or from equation B-7 or B-8
x_3	From knowing M or from equation B-3, B-3A, B-4, B-5, or B-6
Q_s	From equation B-11 or B-12
h_o	From piezometric data or estimated from equations
h_x	From piezometric data or estimated from equations

a. Net head, H. The net head on a levee is the height of water on the riverside above the tailwater or natural ground surface on the landside of the levee. H is usually based on the design or project flood stage but is sometimes based on the net levee grade.

b. Thickness, z and vertical permeability, k_b , of top stratum.

(1) Exploration. The thickness of the top stratum, both riverward and landward of the levee, is extremely important in a seepage analysis. Exploration to determine this thickness usually consists of auger borings with samples taken at 0.91- to 1.52-m (3- to 5-ft) intervals and at every change of material. Boring spacing will depend on the potential severity of the underseepage problem but should be laid out so as to sample the basic geologic features with intermediate borings for check purposes. Landside borings should be sufficient to delineate any significant geological features as far as 152.4 m (500 ft) away from the levee toe. The effect of ditches and borrow areas must be considered.

(2) Transformation. The top stratum in most areas is seldom composed of one uniform material but rather usually consists of several layers of different soils. If the in situ vertical permeability of each soil (k_n) is known, it is possible to transform an overall effective thickness and permeability. However, if good judgment is exercised in selection of these values, a reasonably accurate seepage analysis can be made by using a simplified procedure. Basically this procedure consists of assuming a uniform vertical permeability

for the generalized top stratum equal to the permeability of the most impervious strata and then using the transformation factor given in equation B-1 to determine a corresponding thickness for the entire top stratum.

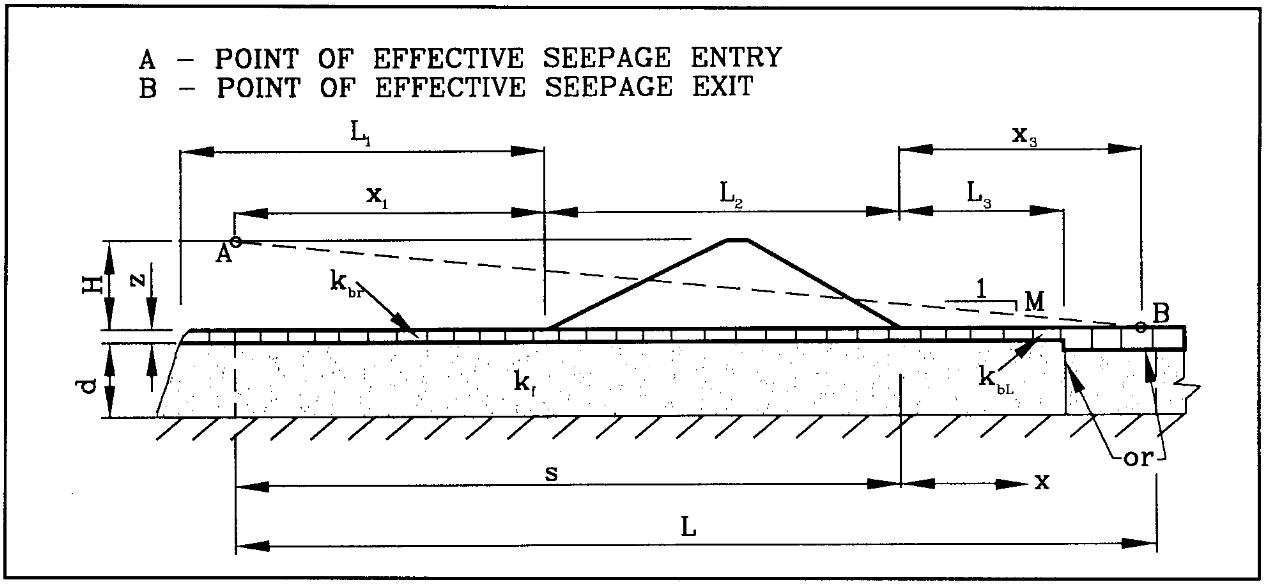


Figure B-1. Illustration of symbols used in Appendix B

$$F_t = \frac{k_b}{k_n} \qquad \text{(B-1)}$$

where F_t = transformation factor.

If the in situ thickness of each soil layer (z_n) is known, the value of corresponding transformed thickness (z_t) can be expressed as

$$z_t = z_n\, F_t = z_n\, \frac{k_b}{k_n} \qquad \text{(B-1a)}$$

The total in situ thickness (z) and total transformed can be expressed as

$$z = \sum_1^n z_n \qquad \text{(B-1b)}$$

$$z_b = \sum_1^n z_t \qquad \text{(B-1c)}$$

Some examples using this procedure are given in Table B-3 and in Figure B-2.

Table B-3
Examples of Transformation Procedure

Strata	Actual Thickness z_n, m (ft)	Actual Permeability cm/sec	$F_t = \frac{K_b}{K_n}$	Transformed Thickness, z_t, m (ft) (k_b = 1x 10^{-4} cm/sec)
Clay	1.52 (5)	1 x 10^{-4}	1	1.52 (5.0)
Sandy silt	2.44 (8)	2 x 10^{-4}	1/2	1.22 (4.0)
Silty sand	1.52 (5)	10 x 10^{-4}	1/10	0.15 (0.5)
	z = 5.48 (18)			z_b = 2.90 (9.5)

A generalized top stratum having a uniform permeability of 1 x10^{-4} cm/sec and 2.9 m (9.5 ft) thick would then be used in the seepage analysis for computation of the length to the effective seepage exit. However, the thickness z_b may or may not be the effective thickness of the landside top stratum z_t that should be used in determining the allowable pressure beneath the top stratum. The transformed thickness of the top stratum for estimating allowable uplift z_t equals the in situ thicknesses of all strata above the base of the least pervious stratum plus the transformed thicknesses of the underlying more pervious top strata. This means that z_b will equal z_t only when the least pervious stratum is at the ground surface. Several examples of this transformation are given in Figure B-2. In making the final determination of the effective thicknesses and permeabilities of the top stratum, the characteristics of the top stratum at least 61 to 91.4 m (200 to 300 ft) landward of the levee must be considered. In addition, certain averaging assumptions are almost always required where soil conditions are reasonably similar. Thin or critical areas should be given considerable weight in arriving at such averages.

c. Thickness d and permeability k_f of pervious substratum. The thickness of the pervious substratum is defined as the thickness of the principal seepage-carrying stratum below the top stratum and above rock or other impervious base stratum. It is usually determined by means of deep borings although a combination of shallow borings and seismic or electrical resistivity surveys may also be employed. The thickness of any individual pervious strata within the principal seepage carrying stratum must be obtained by deep borings. The average horizontal permeability k_f of the pervious substratum can be determined by means of a field pump test on a fully penetrating well or by the use of correlations as shown in Figure 3-5(b) in the main text. For areas where such correlations exist their use will usually result in a more accurate permeability determination than that from laboratory permeability tests. In addition to the methods above, if the total amount of seepage per unit length passing beneath the levee (Q_s), the hydraulic grade line beneath the levee (M) and the thickness of pervious stratum (d) are known, k_f can be estimated from

$$k_f = \frac{Q_s}{M \cdot d} \qquad \text{(B-2)}$$

d. Distance from riverside levee toe to river, L_1 . This distance can usually be estimated from topographic and stratigraphic maps.

e. Base width of levee and berm, L_2 . L_2 can be determined from anticipated dimensions of new levees or by measurement in the case of existing levees.

f. Length of top stratum landward of levee toe, L_3 . This distance can usually be determined from borings, topographic maps, and/or field reconnaissance. In determining this distance careful consideration

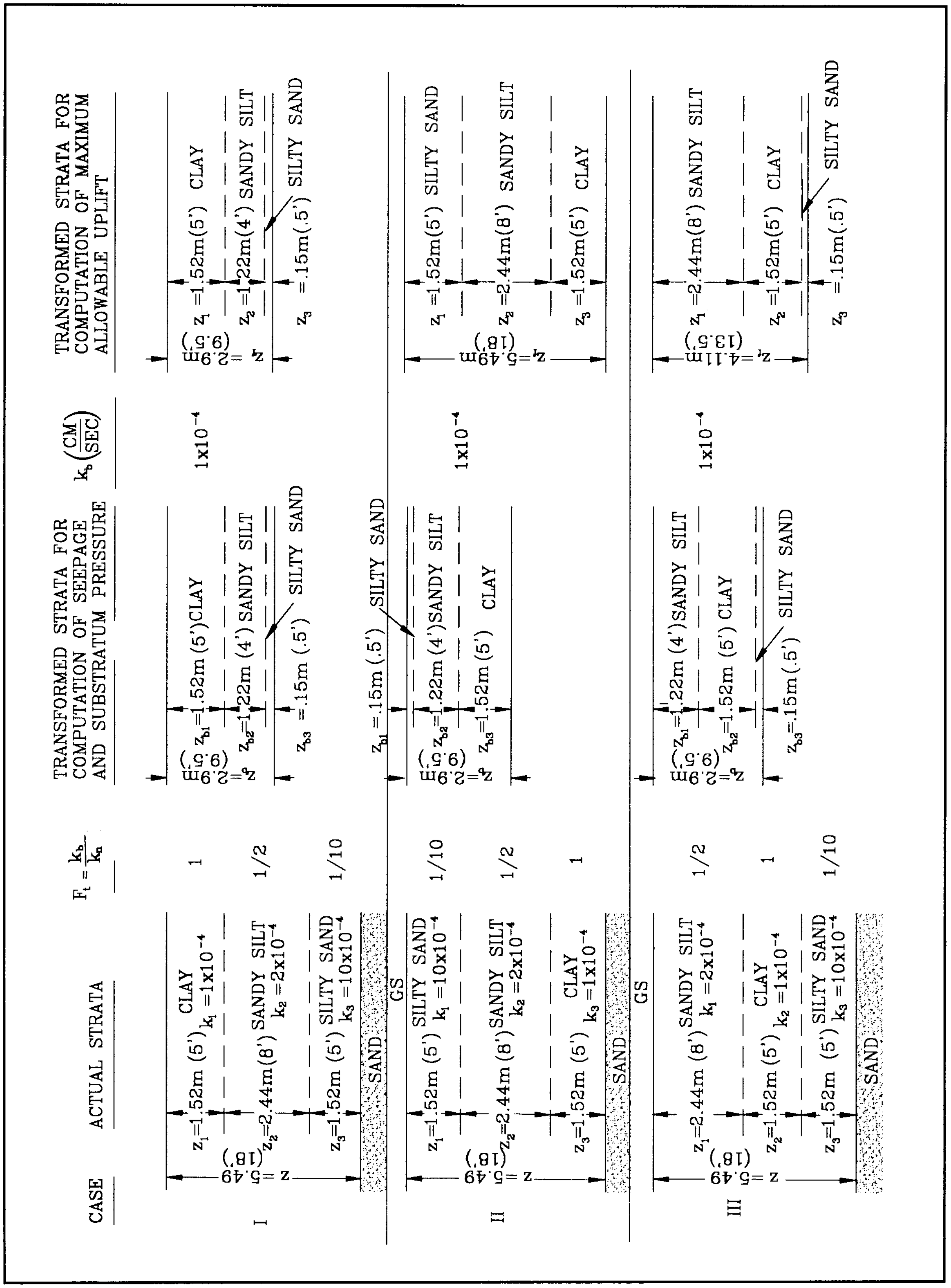


Figure B-2. Transformation of top strata

must be given to any geological feature that may affect the seepage analysis. Of special importance are deposits of impervious materials such as clay plugs which can serve as seepage barriers and if located near the landside toe could force the emergence of seepage at their near edge, thus having a pronounced effect on the seepage analysis.

g. Distance from landside levee toe to effective seepage exit, x_3. The effective seepage exit (point B, Figure B-1) is defined as that point where a hypothetical open drainage face would result in the same hydrostatic pressure at the landside levee toe and would cause the same amount of seepage to pass beneath the levee as would occur for actual conditions. This point is also defined as the point where the hydraulic grade line beneath the levee projected landward with a slope M intersects the groundwater or tailwater. If the length of foundation and top stratum beyond the landside levee toe L_3 is known, x_3 can be estimated from the following equations:

(1) For $L_3 = \infty$

$$x_3 = \frac{1}{c} = \sqrt{\frac{k_f\ z_{bl}\ d}{k_{bl}}} \qquad \text{(B-3)}$$

where

$$c = \sqrt{\frac{k_{bl}}{k_f\ z_{bl}\ d}} \qquad \text{(B-3A)}$$

(2) For L_3 = finite distance to a seepage block

$$x_3 = \frac{1}{c\ \tanh\ (cL_3)} \qquad \text{(B-4)}$$

(3) For L_3 = finite distance to an open seepage exit

$$x_3 = \frac{\tanh\ (cL_3)}{c} \qquad \text{(B-5)}$$

(4) The relationship between z_{bl} and x_3 where L_3 is infinite in landward extent has been computed from equation B-3 and plotted in Figure B-3 for various values of k_f/k_{bl} and assuming d = 100 m or 100 ft. The x_3 value corresponding to values of d other than 100 m or 100 ft can be computed from equation B-6 below:

$$x_3 = (0.1\ \sqrt{d})\ x_3' \qquad \text{(B-6)}$$

where

x_3' is the value of x_3 for d = 100 m or 100 ft

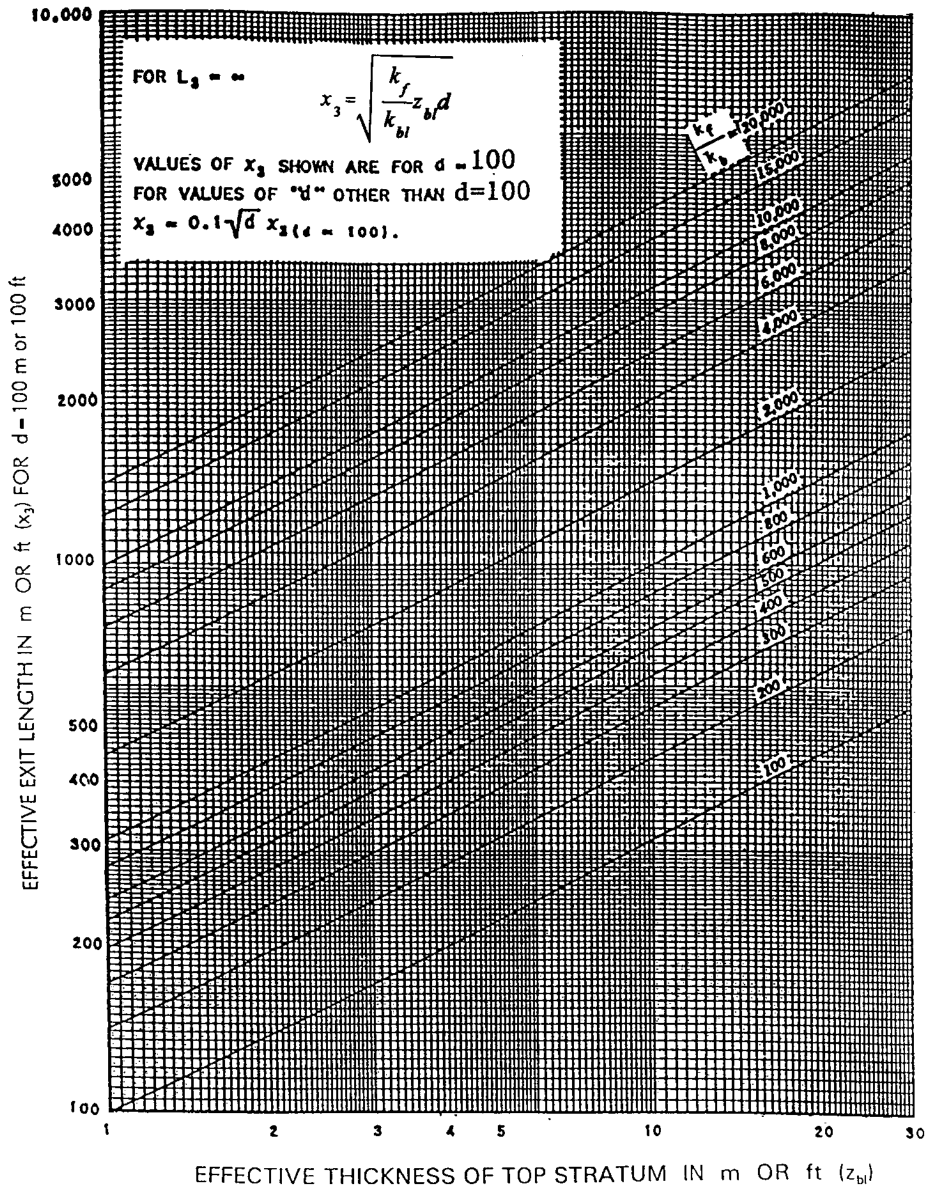


Figure B-3. Effective seepage exit length for $L_3 = \infty$ and d = 100 m or ft

Example: Using Figure B-3, find x_3 for soil with $\frac{k_f}{k_{bl}} = 200$, $z_{bl} = 3.05$ m (10 ft), and d = 45.7 m (150 ft)

Solution: 1 - From Figure B-3 the value x_3 for $\frac{k_f}{k_{bl}} = 200$, $z_{bl} = 3.05$ m (10 ft). Then for $z_{bL} = 3.05$ m and d = 100 m, $x_3' = 246$ m; or $z_{bL} = 10$ ft and d = 100 ft, $x_3' = 450$ ft

2 - Apply Equation B-6 to determine x_3 for d = 45.7 m (150 ft)

$$x_3 = 0.1 \sqrt{45.7}\; x_3'$$

$$x_3 = (0.1)\,(6.76)\,(246) = 167\ m$$

or

$$x_3 = 0.1 \sqrt{d}\ (450) = 0.1\sqrt{150}\ (450) = 551\ ft$$

(5) If L_3 is a finite distance either to a seepage block or an open seepage exit, the effective exit length x_3 can be computed by using equation B-4 or B-5 or by multiplying x_3 (for $L_3 = \infty$) by a factor obtained from Figure B-4.

h. Distance from effective source seepage entry to riverside levee toe, x_1. The effective source of seepage entry into the pervious substratum (point A in Figure B-1) is defined as that line riverward of the levee where a hypothetical open seepage entry face fully penetrating the pervious substratum and with an impervious top stratum between this line and the levee would produce the same flow and hydrostatic pressure beneath and landward of the levee as will occur for the actual conditions riverward of the levee. It is also defined as that line or point where the hydraulic grade line beneath the levee projected riverward with a slope M intersects the river stage.

(1) If the distance to the river from the riverside levee toe L_1 is known and no riverside borrow pits or seepage blocks exist, x_1 can be estimated from the following equation:

$$x_1 = \frac{\tanh\ cL_1}{c} \qquad \text{(B-7)}$$

(2) If a seepage block (usually a wide, thick deposit of clay) exists between the riverside levee toe and the river so as to prevent any seepage entrance into the pervious foundation beyond that point, x_1 can be estimated from the following equation:

$$x_1 = \frac{1}{c\ \tanh\ cL_1} \qquad \text{(B-8)}$$

where L_1 equals distance from riverside levee toe to seepage block and c is from equation B-3A.

i. Critical gradient for landside top stratum, i_c. The critical gradient is defined as the gradient required to cause boils or heaving (flotation) of the landside top stratum and is taken as the ratio of the submerged or buoyant unit weight of soil ã′ comprising the top stratum and the unit weight of water $ã_w$ or

$$i_c = \frac{ã'}{ã_w} = \frac{G_S - 1}{1 + e} \qquad \text{(B-9)}$$

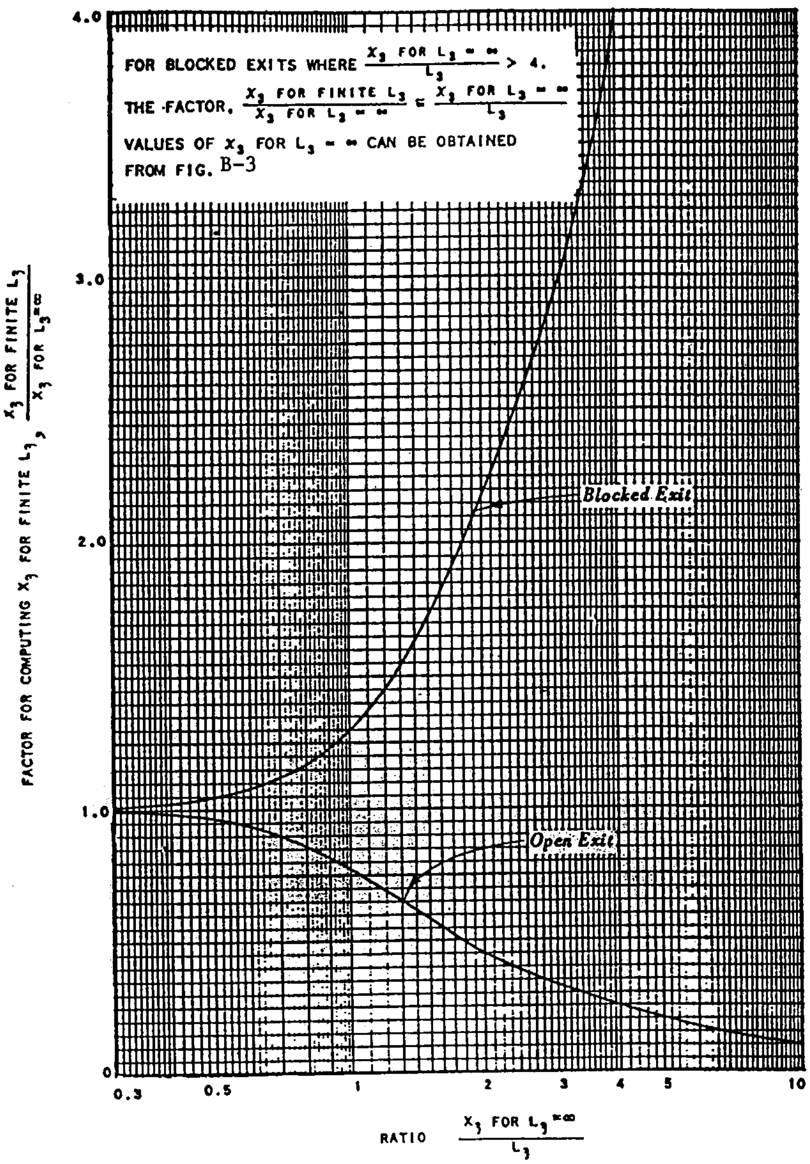


Figure B-4. Ratio between x_3 for blocked or open exits and x_3 for $L_3 = \infty$

where

G_S = specific gravity of soil solids
e = void ratio

j. Slope of hydraulic grade line beneath levee, M. The slope of the hydraulic grade line in the pervious substratum beneath a levee can best be determined from readings of piezometers located beneath the levee where the seepage flow lines are essentially horizontal and the equipotential lines vertical. If such readings during high water are available, M can be determined from the following relation:

$$M = \frac{Äh}{} \qquad \text{(B-10a)}$$

where

Äh = the difference in piezometer readings
= the horizontal distance between piezometers

This relationship is not valid, however, until artesian flow conditions have developed beneath the levee. If no piezometer readings are available, as in the case for new levee design, M must be determined by exit points and first establishing the effective seepage entrance and then connecting these points with a straight line, the slope of which is M. For new levees M is expressed as

$$M = \frac{H}{x_1 + L_2 + x_3} \qquad \text{(B-10b)}$$

B-5. Computation of Seepage Flow and Substratum Hydrostatic Pressures

a. General

(1) Seepage. For a levee underlain by a pervious foundation, the natural seepage per unit length of levee, Q_S, can be expressed by the general equation B-11.

$$Q_s = \mathcal{S} k_f H \qquad \text{(B-11)}$$

where

$\mathcal{S}$ = shape factor

This equation is valid provided the assumptions upon which Darcy's law is based are met. The mathematical expressions for the shape factor $\mathcal{S}$ (subsequently given in this appendix) depend upon the dimensions of the generalized cross section of the levee and foundation, the characteristics of the top stratum both riverward and landward of the levee, and the pervious substratum. Where the hydraulic grade line M is known from piezometer readings, the quantity of underseepage per unit length of the levee can be determined from equation B-12 as

$$Q_s = M k_f d \qquad \text{(B-12)}$$

(2) Excess hydrostatic head beneath the landside top stratum.

(a) The excess hydrostatic head h_o beneath the top stratum at the landside levee toe is related to the net head on the levee, the dimensions of the levee and foundation, permeability of the foundation, and the character of the top stratum both riverward and landward of the levee. The head h_o can be expressed as a function of the net head H and the geometry of the piezometric line as subsequently shown.

(b) The head h_x beneath the top stratum at a distance x landward from the landside levee toe can be expressed as a function of the net head H and the distance x although it is more conveniently related to the head h_o at the levee toe. When h_x is expressed in terms of h_o it depends only upon the type and thickness of the top stratum and pervious foundation landward of the levee; the ratio h_x/h_o is thus independent of riverward conditions.

(c) Expressions for $\mathcal{S}$, h_o, and h_x are discussed in the following paragraphs.

b. Various underseepage flow and top substratum conditions.

Case 1 - No Top Stratum. Where a levee is founded directly on pervious materials and no top stratum exists either riverward or landward of the levee (Figure B-5a), the seepage Q_S can be obtained from equation B-11 in which

$$\mathcal{S} = \frac{d}{L_2 + 0.86d} \tag{B-13}$$

The excess hydrostatic head landward of the levee is zero and $h_o = h_x = 0$. The severity of such a condition in nature is governed by the exit gradient and seepage velocity that develop at the landside levee toe which can be estimated from a flow net compatible with the value of $\mathcal{S}$ computed from Equation B-13. The maximum allowable exit gradient should be 0.5.

Case 2 - Impervious Top Stratum Both Riverside and Landside. This case is found in nature where the levee is founded on thick (z_{bl} > 4.58 m (15 ft)) deposits of clay or silts with clay strata. For such a condition little or no seepage can occur through the landside top stratum.

a. If the pervious substratum is blocked landward of the levee, no seepage occurs beneath the levee and $Q_s = 0$. The head beneath the levee and the landside top stratum is equal to the net head at all points so that $H = h_o = h_x$.

b. If the top stratum is impervious between the levee and river and has a length L_1, and if an open seepage exit exists in the impervious top stratum at some distance L_3 from the landside toe (i.e., L_3 is not infinite as shown in Figure B-5b), the distance from the landside toe of the levee to the effective seepage entry (river, borrow pit, etc.) is $S = L_1 + L_2$ and

$$\mathcal{S} = \frac{d}{L_1 + L_2 + L_3} \tag{B-14}$$

The heads h_o and h_x can be computed from

$$h_o = H\left(\frac{L_3}{L_1 + L_2 + L_3}\right) \tag{B-15}$$

$$h_x = h_o\left(\frac{L_3 - x}{L_3}\right) \text{ for } x \quad L_3 \tag{B-16}$$

$$h_x = 0 \text{ for } x \quad L_3$$

Case 3 - Impervious Riverside Top Stratum and No Landside Top Stratum. This condition may occur naturally or where extensive landside borrowing has taken place resulting in removal of all impervious material landward of the levee for a considerable distance. Seepage can be computed utilizing Equation B-11 and the following shape factor

$$\mathcal{S} = \frac{d}{L_1 + L_2 + 0.43d} \qquad \text{(B-17)}$$

The excess head at the top of the sand landward of the levee is zero and the danger from piping must be evaluated from the upward gradient obtained from a flow net. This case is shown in Figure B-5c.

Case 4 - Impervious Landside Top Stratum and No Riverside Top Stratum. This is a more common case than Case 3, occurring when extensive riverside borrowing has resulted in removal of the riverside impervious top stratum (Figure B-5d). For this condition the seepage is computed from Equation B-11 utilizing the shape factor given in Equation B-18 below; the heads h_o and h_x can be computed from Equations B-19 and B-20, respectively.

$$\mathcal{S} = \frac{d}{0.43d + L_2 + L_3} \qquad \text{(B-18)}$$

$$h_o = H\left(\frac{L_3}{0.43d + L_2 + L_3}\right) \qquad \text{(B-19)}$$

$$h_x = h_o\left(\frac{L_3 - x}{L_3}\right) \qquad \text{(B-20)}$$

Case 5 - Semipervious Riverside Top Stratum and No Landside Top Stratum. The same equation for the shape factor as was used in Case 3 can be applied to this condition provided x_1 is substituted for L_1 as follows:

$$\mathcal{S} = \frac{d}{x_1 + L_2 + 0.43d} \qquad \text{(B-21)}$$

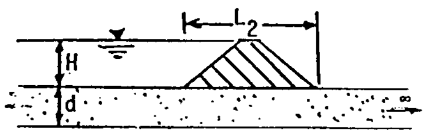
Since no landside top stratum exists, $h_o = h_x = 0$. This case is illustrated in Figure B-6a.

Case 6 - Semipervious Landside Top Stratum and No Riverside Top Stratum. The same equations for the shape factor and heads beneath the landside top stratum that are used for Case 4 are applicable to this case provided x_3 is substituted for L_3 (Figure B-6b). These equations are as follows:

$$\mathcal{S} = \frac{d}{0.43d + L_2 + x_3} \qquad \text{(B-22)}$$

$$h_o = H\left(\frac{x_3}{0.43d + L_2 + x_3}\right) \qquad \text{(B-23)}$$

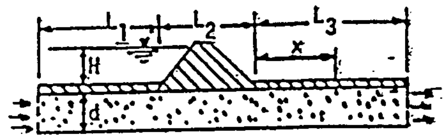
a. CASE 1 - No top stratum



$$\mathcal{S} = \frac{d}{L_2 + 0.86d}$$

$$h_o = h_x = 0$$

b. CASE 2 - Impervious topstratum both riverside and landside



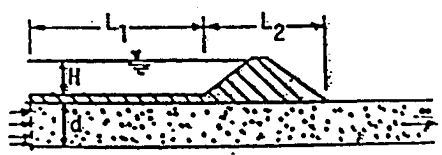
$$\mathcal{S} = \frac{d}{L_1 + L_2 + L_3}$$

$$h_o = H\left(\frac{L_3}{L_1 + L_2 + L_3}\right)$$

$$h_x = h_o\left(\frac{L_3 - x}{L_3}\right) \text{ for } x < L_3$$

$$h_x = 0 \text{ for } x > L_3$$

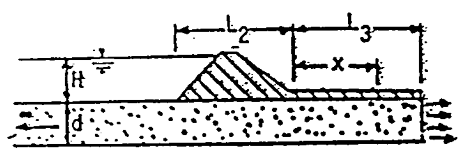
c. CASE 3 - Impervious riverside top stratum & no landside top stratum



$$\mathcal{S} = \frac{d}{L_1 + L_2 + 0.43d}$$

$$h_o = h_x = 0$$

d. CASE 4 - Impervious landside top stratum & no riverside top stratum



$$\mathcal{S} = \frac{d}{0.43d + L_2 + L_3}$$

$$h_o = H\left(\frac{L_3}{0.43d + L_2 + L_3}\right)$$

$$h_x = h_o\left(\frac{L_3 - x}{L_3}\right)$$

BASIC DEFINITIONS AND EQUATIONS

HEAD BENEATH TOP STRATUM AT LANDSIDE LEVEE TOE . . . . h_o

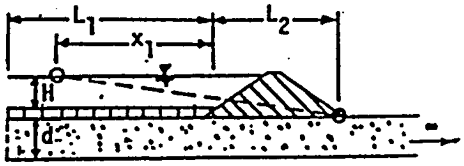
HEAD BENEATH TOP STRATUM AT DISTANCE x FROM LANDSIDE LEVEE TOE . . . h_x

SHAPE FACTOR FOR USE IN SEEPAGE EQUATION . . . . $\mathcal{S}$

SEEPAGE PER UNIT LENGTH OF LEVEE . . . . $Q_s = \mathcal{S} k_f H$

Figure B-5. Equations for computation of underseepage flow and substratum pressures for cases 1 through 4

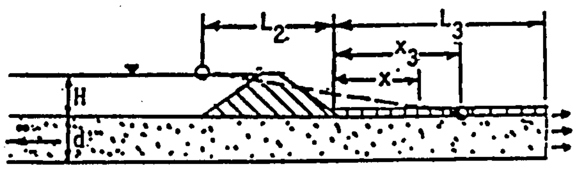
a. CASE 5 - Semipervious riverside top stratum and no landside top stratum



$$\mathcal{S} = \frac{d}{x_1 + L_2 + 0.43d}$$

$$h_o = h_x = 0$$

b. CASE 6 - Semipervious landside top stratum and no riverside top stratum



$$\mathcal{S} = \frac{d}{0.43d + L_2 + x_3}$$

$$h_o = H\left(\frac{x_3}{0.43d + L_2 + x_3}\right)$$

$$h_x = h_o\left(\frac{x_3 - x}{x_3}\right)$$

Figure B-6. Equations for computation of underseepage flow and substratum pressures for cases 5 and 6

$$h_x = h_o\left(\frac{x_3 - x}{x_3}\right) \tag{B-24}$$

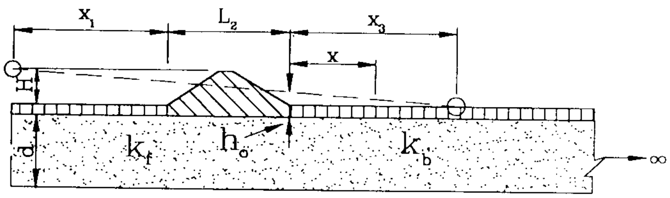
Case 7 - Semipervious Top Strata Both Riverside and Landside. Where both the riverside and landside top strata exist and are semipervious (Figure B-7), the quantity of underseepage can be computed from equation B-11 where $\mathcal{S}$ is defined in Equation B-25.

$$\mathcal{S} = \frac{d}{x_1 + L_2 + x_3} \tag{B-25}$$

The head beneath the top stratum at the landside toe of the levee is expressed by

$$h_o = H\left(\frac{x_3}{x_1 + L_2 + x_3}\right) \tag{B-26}$$

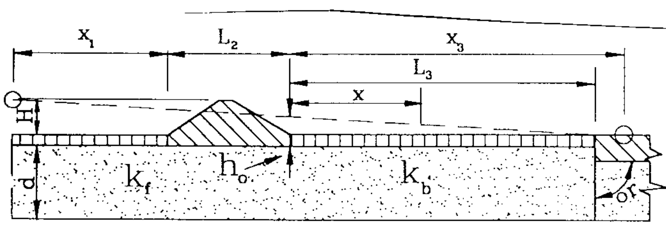
The equations above are valid for all conditions where the landside top stratum is semipervious. However, the head h_x beneath the semipervious top stratum depends not only on the head h_o but also on conditions landward of the levee. Expressions are given below for typical conditions encountered landward of levees.



$x_3 = \frac{1}{c}$

$h_x = h_o \ e^{-cx}$

a. **$L_3 = \infty$**

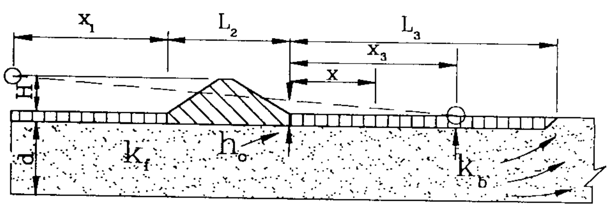


$x_3 = \frac{1}{c \tanh(cL_3)}$

$h_x = h_o \frac{\cos h\, c(L_3 - x)}{\cos h\ cL_3}$

$h_x \ (\text{for } x = L_3) = \frac{h_o}{\cos h\ cL_3}$

b. **L_3 is finite to a seepage block**



$x_3 = \frac{\tan h(cL_3)}{c}$

$h_x = h_o \frac{\sin h\ c(L_3 - x)}{\sin h\ cL_3}$

$h_x \ (\text{for } x = L_3) = 0$

c. **L_3 is finite to an open seepage exit**

<u>BASIC DEFINITIONS AND EQUATIONS</u>

SEEPAGE PER UNIT LENGTH OF LEVEE...................................... $Q_s = \$ k_f H = \dfrac{dk_f H}{x_1 + L_2 + x_3}$

HEAD BENEATH TOP STRATUM AT LANDSIDE LEVEE TOE........... $h_o = \dfrac{Hx_3}{x_1 + L_2 + x_3}$

THE FACTOR.. $c = \sqrt{\dfrac{kbL}{k_f z_b d}}$

Figure B-7. Equations for computation of underseepage and substratum pressures for Case 7

(1) For $L_3 = \infty$

$$h_x = h_o \; e^{-cx} \tag{B-27}$$

where

e = 2.718

(2) For L_3 = a finite distance to a seepage block

$$h_x = h_o \frac{\cosh \, c(L_3 - x)}{\cosh \, cL_3} \tag{B-28}$$

and

$$h_x \; (at \;\; x = L_3) = \frac{h_o}{\cosh \, cL_3} \tag{B-29}$$

(3) For L_3 = a finite distance to an open seepage exit

$$h_x = h_o \frac{\sinh \, c(L_3 - x)}{\sinh \, cL_3} \tag{B-30}$$

and

$$h_x \; (at \;\; x = L_3) = 0 \tag{B-31}$$

(4) Values of c and h_o in Equations B-27 through B-30 are as follows:

$$c = \sqrt{\frac{k_{bl}}{k_f \; z_{bl} \; d}} \quad , \qquad h_o = H \frac{x_3}{x_1 + L_2 + x_3} \tag{B-32}$$

(5) In order to simplify the determination of h_x for various values of x, the relationship between h_x/h_o and x/x_3 is plotted in Figure B-8 for $L_3 = \infty$ and for various values of x_3/L_3 for both a seepage block and an open seepage exit. The procedure for determining h_x using Figure B-8 can be summarized as follows:

a. Determine x_1, L_2, x_3 and the head h_o at the landside toe of the levee.

b. For the given distance x where h_x needs to be determined find the ratios x/x_3 and x_3/L_3, then enter the appropriate graph in Figure B-8 to read the corresponding value of h_x/h_o.

c. Knowing the ratio h_x/h_o and the value of h_o compute h_x.

(6) Values of h_o and h_x resulting from the equations above are actually hydrostatic heads at the middle of the pervious substratum; where the ratio k_f/k_b is less than 100 to 500, values of h_o and h_x immediately

beneath the top stratum will be slightly less than those computed because of the head loss resulting from upward seepage through the sand stratum.

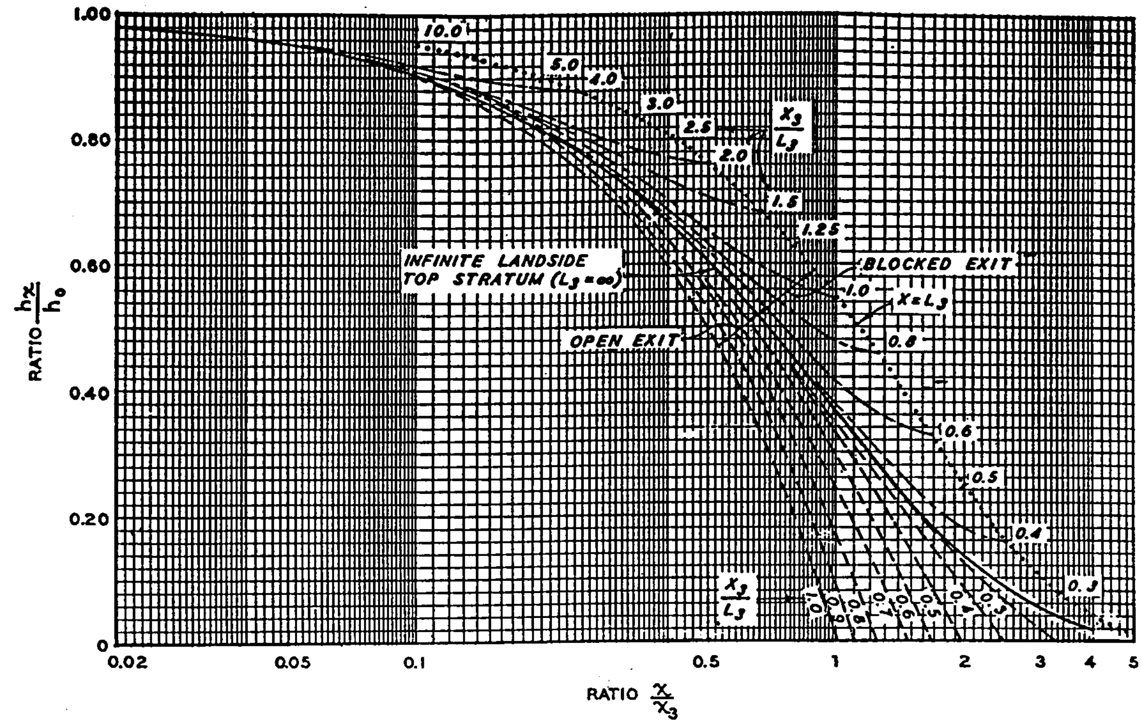


Figure B-8. Ratio between head landward of levee and head at landside toe of levee for levees founded on semipervious top stratum underlain by a pervious substratum