Appendix C
Notes on Derivation and Use of Hydraulic Properties by the Alpha Method

C-1. General

The Alpha method for determining the local boundary shear and composite roughness is applicable to uniform and gradually varied flow problems. Computations for effective average channel roughness k with and without considering the energy correction factor are included as well as computations for Manning's n. The necessary basic equations and a computation procedure are given in the paragraphs that follow. Illustrations of the Alpha method applied to the effective channel roughness problem are given in Plates C-1 through C-4.

C-2. Basic Procedure and Equations

a. The cross section (Plate C-1) is divided into subsections bounded by vertical lines extending from water surface to the wetted perimeter. The mean velocity in the vertical of the subsection is given by V_n and the subsection discharge by $V_n A_n$. The integer subscript n defines the channel subsection. As explained in paragraph 6-5 of Chow (1959),[1] a simplifying assumption becomes necessary. It is assumed that the energy grade line has the same slope across the entire cross section, that S in the familiar Chezy equation ($V = C(RS)^{1/2}$) is constant at each subsection, and that the following proportion may be written

$$V_n :: \left(CR^{1/2}\right)_n \tag{C-1}$$

where C is Chezy's coefficient and R is the hydraulic radius.

b. The resistance equation for hydraulically rough channels (paragraph 2-2(c)) is

$$C = 32.6 \log_{10} \frac{12.2R}{k} \tag{C-2}$$

[1] References cited in this appendix are listed in Appendix A.

where

C = Chezy's coefficient

R = hydraulic radius, ft

k = equivalent roughness dimension, ft

This equation is plotted in Plate C-2.

c. As $(CR^{1/2})_n$ is proportional to V_n, then $(CR^{1/2})_n A_n$ is proportional to Q_n.[2] From this the following equations are derived

$$Q_n = \frac{Q_T \left(CR^{1/2}\right)_n A_n}{\sum \left[\left(CR^{1/2}\right)_i A_i\right]} \tag{C-3}$$

$$V_n = \frac{Q_T \left(CR^{1/2}\right)_n}{\sum \left[\left(CR^{1/2}\right)_i A_i\right]} \quad or \quad \frac{Q_n}{A_n} \tag{C-4}$$

$$\left(CR^{1/2}\right)_{\text{mean}} = \frac{\sum \left[\left(CR^{1/2}\right)_i A_i\right]}{\sum A_i} \tag{C-5}$$

$$S = \frac{\bar{V}^2}{\left[\left(CR^{1/2}\right)_{\text{mean}}\right]^2} \tag{C-6}$$

$$\bar{R} = \frac{\sum \left[R_i \left(CR^{1/2}\right)_i A_i\right]}{\sum \left[\left(VR^{1/2}\right)_i A_i\right]} \tag{C-7}$$

where

Q_n = discharge in subsection, cfs

Q_T = total discharge, cfs

[2] The subscript i assumes all values of n.

A = cross-sectional area, ft^2

$\bar{V}$ = flow velocity in subsection, fps

$\bar{R}$ = hydraulic radius of subsection

C-3. Backwater Computation

a. All the cross-section hydraulic parameters necessary for backwater computations are computed in Plates C-2 and C-3. Computing the same parameters at several water-surface elevations and plotting the results permits ready interpolation for intermediate values. The method is programmed for digital computer use if manual computations for a particular project are too time consuming.

b. The boundary and hydraulic characteristics of a channel reach are assumed to be those obtained by averaging the conditions existing at each end of the reach. This procedure implies that the roughness dimensions k assigned to the upstream and downstream sections extend to the midsection of the reach. Therefore, it is important that the reach limits be carefully selected. Two different sets of subsection roughness values should be assigned in cases where the boundary condition changes abruptly such as at the beginning or end of an improved reach. One set of values would apply in the improved reach and the other in the natural channel.

C-4. Roughness Relation

The roughness dimension k may be taken as equivalent spherical diameter of the average size bed material when the hydraulic losses in the flow regime are attributable to friction alone. In a flow regime where hydraulic losses in addition to friction are present, k may still be used if the losses result in a reasonably uniform slope of the energy grade line. In this case, k will be larger dimensionally than the equivalent spherical diameter of the average size bed material. As Chezy C and Manning's n are equatable $(C/1.486 = R^{1/6}/n)$, k may be determined from a knowledge of Manning's coefficient n. While k remains fairly constant with changing R, n varies with the onesixth power of R. Therefore, it is better to extrapolate from known conditions to unknown by the use of k rather than n. The k must be evaluated for each subsection. Subsections should be chosen with this in mind so that differing bed materials or bed conditions producing frictionlike losses, such as ripples, dunes, or other irregularities will appear in separate subsections. Hydraulic losses tending to cause breaks in the energy grade line, such as expansion and contraction, should be evaluated separately. Computations are presented in Plate C-4 showing the use of the Alpha computation results for determining an effective channel k value and the relation between k and n.

C-5. Energy Correction Factor

The velocity head correction factor (Brater and King 1976) is expressed as

$$\alpha = \frac{1}{AV^3} \int_o^A V_x^3 \, dA \tag{C-8}$$

where

α = velocity head correction factor

V = mean velocity of the section

V_x = mean velocity in the vertical at horizontal location x throughout the cross section

The mean velocity may be expressed

$$V = \frac{\int_o^A V_x \, dA}{A} \tag{C-9}$$

Substituting Equation C-9 in C-8 yields

$$\alpha = \frac{A^2 \int_o^A V_x^3 \, dA}{\left(\int_o^A V_x \, dA \right)^3} \tag{C-10}$$

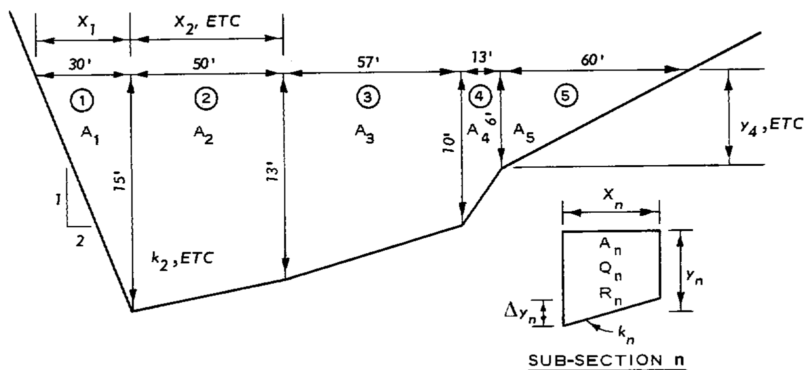
Substituting the relation given by Equation C-1 into Equation C-10 yields

$$\alpha = \frac{A^2 \int_o^A \left(CR^{1/2}\right)_x^3 \, dA}{\left[\int_o^A \left(CR^{1/2}\right)_x \, dA\right]^3} \tag{C-11}$$

or

$$\alpha = \frac{A^2 \sum \left[\left(CR^{1/2}\right)_n^3 A_n\right]}{\left[\sum \left(CR^{1/2}\right)_i A_i\right]^3} \tag{C-12}$$

Computations illustrating the application of the Alpha method for determining the energy correction factor α are given in Plate C-4. In addition, the effect of the energy correction factor on the apparent average channel roughness value is shown.



CHANNEL GEOMETRY

$Q_T = \Sigma Q_n = 20{,}000$ CFS

SUBSECTION	1	2	3	4	5
X_n, FT	30.0	50.0	57.0	13.0	60.0
y_n, FT	7.5	14.0	11.5	8.0	3.0
AREA A_n, FT^2	225.0	700.0	655.0	104.0	180.0
k_n	1.33	2.0	2.5	3.0	4.0
P_n	33.5	50.0	57.0	13.6	60.3
HYDRAULIC RADIUS R_n	6.7	14.0	11.5	7.7	3.0
C (PLATE C-2)	59	63	57	48	31
$(CR^{1/2})_n$	153	236	193	134	54
$(CR^{1/2})_n A_n \times 10^{-3}$	34.4	165.2	126.4	13.9	9.7
$(CR^{1/2})^3_n A_n \times 10^{-9}$	0.81	9.20	4.71	0.25	0.03
$(CR^{3/2})_n A_n \times 10^{-6}$	0.23	2.31	1.45	0.11	0.03

$A_n = X_n y_n$, WETTED PERIMETER $P_n = \left[X_n^2 + (\Delta y_n)^2\right]^{1/2}$

$A_n = \frac{A_n}{P_n}$, k_n = EFFECTIVE ROUGHNESS HEIGHT

$Q_n = A_n V_n$, $A = \Sigma A_i$, $Q_T = \Sigma Q_i$

GEOMETRIC PROPERTIES

ALPHA METHOD
GEOMETRIC COMPUTATIONS

PLATE C-1

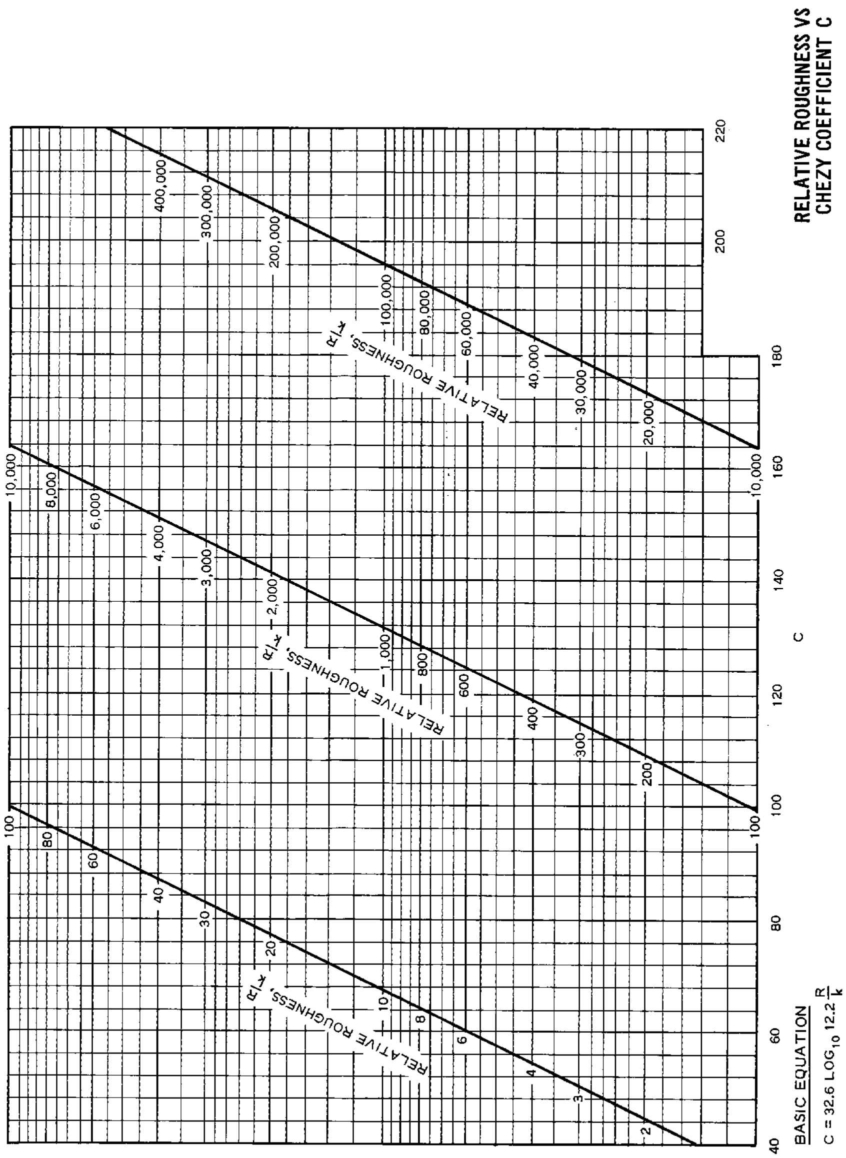

RELATIVE ROUGHNESS VS
CHEZY COEFFICIENT C
BASIC EQUATION
C = 32.6 LOG10 12.2 R/k
RELATIVE ROUGHNESS, R/k
RELATIVE ROUGHNESS, R/k
RELATIVE ROUGHNESS, R/k
C
40
60
80
100
120
140
160
180
200
220
2
3
4
6
8
10
20
30
40
60
80
100
200
300
400
600
800
1,000
2,000
3,000
4,000
6,000
8,000
10,000
20,000
30,000
40,000
60,000
80,000
100,000
200,000
300,000
400,000

PLATE C-2

1. CALCULATE THE AVERAGE VELOCITY, $\overline{V}$.

$\overline{V} = Q_T/A$

$\overline{V} = (20{,}000)/(1864.0) = 10.7$ FPS

2. CALCULATE THE DISCHARGE THROUGH EACH SUB-SECTION, Q_n.

$$Q_n = \frac{Q_T (CR^{1/2})_n A_n}{\Sigma[(CR^{1/2})_i A_i]} = \frac{20{,}000 (CR^{1/2})_n A_n}{349{,}600}$$

$Q_1 = 0.0572(34400) = 1968$ CFS
$Q_2 = 0.0572(165200) = 9449$
$Q_3 = 0.0572(126400) = 7230$
$Q_4 = 0.0572(13900) = 795$
$Q_5 = 0.0572(9700) = 555$

$\Sigma Q_n = 19{,}997$

3. CALCULATE THE VELOCITY THROUGH EACH SUB-SECTION

$$V_n = \frac{Q_n}{A_n}$$

$V_1 = (1968)/(225.0) = 8.7$ FPS
$V_2 = (9449)/(700.0) = 13.5$
$V_3 = (7230)/(655.0) = 11.0$
$V_4 = (795)/(104.0) = 7.6$
$V_5 = (555)/(180.0) = 3.1$

4. CALCULATE THE MEAN SLOPE OF ENERGY GRADE LINE, S.

$$S = \frac{(\overline{V})^2}{(CR^{1/2})^2_{MEAN}}$$

$$(CR^{1/2})_{MEAN} = \frac{\Sigma[(CR^{1/2})_i A_i]}{A} = \frac{349{,}600}{1864.0} = 188$$

$$S = (10.7)^2/(188)^2 = 0.00324$$

5. CALCULATE THE MEAN HYDRAULIC RADIUS, $\overline{R}$.

$$\overline{R} = \frac{\Sigma[(CR^{3/2})_i A_i]}{\Sigma[(CR^{1/2})_i A_i]}$$

$\overline{R} = (4.13 \times 10^6)/(0.3496 \times 10^6) = 11.8$ FT

6. CALCULATE THE AVERAGE SHEAR FORCE $\overline{\tau}_0$

$\overline{\tau}_0 = \gamma \overline{R} S$
$= (62.5)(11.8)(0.00324) = 2.39$ LB/FT^2

ALPHA METHOD
HYDRAULIC PROPERTIES

PLATE C-3

1. CALCULATE ENERGY CORRECTION FACTOR

$$\alpha = \frac{A^2 \Sigma \left[(CR^{1/2})_i^3 \; A_i \right]}{\left[\Sigma (CR^{1/2})_i \;\; ^3 \;\; A_i \right]}$$

FROM PLATE C-1

$$A^2 = (\Sigma A_i)^2 = (1864.0)^2 = 3.47 \times 10^6$$

$$\Sigma \left[(CR^{1/2})_i^3 \; A_i \right] = 15.00 \times 10^9$$

$$\left[\Sigma \; (\; CR^{1/2})_i \; A_i \right]^3 = (349.6 \times 10^3)^3 = 42.7 \times 10^{15}$$

$$\alpha = \frac{(3.474 \times 10^6)(15.00 \times 10^9)}{42.7 \times 10^{15}} = 1.22$$

2. EFFECTIVE k (α NEGLECTED)

$$C^2 = 32.6 \; LOG_{10} \; 12.2 \frac{R}{k} = \frac{\gamma \overline{V}^2}{\overline{\tau}_0}$$

FOR $\overline{V} = 10.7$ AND $\overline{\tau}_0 = 2.39$ (PLATE C-3)

$$C = \frac{\left(62.5 \times \overline{10.7}^2\right)^{1/2}}{2.39} = 54.7$$

FOR C = 54.7, $\frac{R}{k} = 3.9$ (PLATE C-2)

FOR $\overline{R} = 11.8$ (PLATE C-3)

k = 3.03 FT

3. EFFECTIVE k (α CONSIDERED)

$$\alpha \frac{\overline{V}^2}{2g} = \frac{(1.22)(10.7)^2}{64.4} = 2.17 \text{ FT}$$

$$V^1 = (64.4 \times 2.17)^{1/2} = 11.8 \text{ FPS}$$

$$C = \left[\frac{(62.5)\,(11.8)^2}{2.39} \right]^{1/2} = 60.3$$

$\frac{R}{k} = 5.8$ (PLATE C-2)

k = 2.03 FT FOR $\overline{R} = 11.8$ FT

4. CALCULATE MANNING'S n (α NEGLECTED)

$$n = \frac{1.486 \; \overline{R}^{2/3} \; S^{1/2}}{\overline{V}} = \frac{(1.486)(11.8)^{2/3}(0.00324)^{1/2}}{10.7}$$

$$= 0.041$$

ALPHA METHOD
BACKWATER COMPUTATION DATA

PLATE C-4