Chapter 5
Snowmelt—Energy Budget Solutions

5-1. Overview

This chapter will present one of the two basic approaches to computing snowmelt, that of using energy budget equations. With this method an attempt is made to make the solution as physically based as practicable by incorporating into snowmelt equations factors such as solar radiation, wind, and long-wave radiation exchange. The second basic method, called the temperature index solution, will be covered in Chapter 6. In that more simplified approach, air temperature is assumed to be a representative index of all energy sources so that it can be used as the sole independent variable in calculating snowmelt. In Chapters 5 and 6, discussion and guidance will be presented on the appropriate usage of either of these two approaches, and Chapter 10 contains examples of applications of both methodologies.

a. Background and perspective. Researchers have, for a long time, identified the basic energy sources involved in producing snowmelt as discussed in Chapter 2. Among the earliest of these were the USACE snow investigation studies, which were aimed primarily at providing procedures for deriving maximum design floods. These studies led to the development of several generalized energy budget equations, which will be presented in this chapter, along with a summary of the technical concepts embodied in the equations. Seen from today's perspective, the USACE energy budget equations remain as viable tools that are still referenced in textbooks, handbooks, and technical papers. More recent research—see compilations by Male and Gray (1981) and Gray and Prowse (1992)—has tended to emphasize theoretical aspects of snowmelt. Even so, an empirical aspect is often present with field and laboratory experimentation being involved. The USACE equations presented in *Snow Hydrology* generally take a further step away from the theoretical by making additional assumptions, eliminating the dependence on hard-to-obtain data where possible, and combining empirical factors for simplicity. The result is that they are reasonably easy to use in engineering applications. Recent literature typically omits this step; thus, the equations remain useful as an additional bridge between the theoretical and the practical. The equations should not be used, however, without knowledge of the basic technical concepts involved; remember that they were developed from experimental data from three field sites representing specific climatic and topographical regimes.

b. Applications. As noted above, the generalized snowmelt equations were developed primarily to derive the maximum hypothetical design floods in snow regimes. That does not preclude their use in other applications, however, and in fact the equations are included in both the HEC-1 and SSARR models for general use. However, the use of meteorological variables such as solar radiation, dew point, and wind velocity generally preclude their use for real-time forecasting or perhaps for early phases of planning and engineering studies. The equations are very useful for gaining an introductory understanding of the basic principles of snowmelt and can be useful in guiding the application of the temperature index method for forecasting and analysis. Their use in developing hypothetical design floods is quite appropriate and feasible.

5-2. Basis for Equations

a. Overview. Chapter 2 describes the fundamental processes involved in the melting of snow, and Equation 2-1 expresses the basic energy balance equation appropriate for computing snowmelt runoff. There are six external sources of heat energy represented in that equation, and these must be accounted for one way or another in developing applied snowmelt equations. The following discussion will briefly summarize the theoretical principles associated with each of these components, following up from the general description in Chapter 2, then describe the assumptions reflected in the adapted relationships that make up the generalized equations presented in Paragraphs 5-3 and 5-4. The basic source of backup information is *Snow Hydrology* (USACE 1956), unless otherwise noted. For a background on some of the basic physics principles involved, see Appendix C. Appendix D contains background on basic meteorological relationships pertaining to snow hydrology, including some pertinent charts taken from *Snow Hydrology*.

b. Units. The equations in this chapter will be presented in both SI and English units. For the discussion on the sources of the generalized energy budget equations, the SI convention will be followed as much as possible, as was done in Chapter 2. If the reader refers to modern textbooks on physics and meteorology on this subject, the SI convention would be used exclusively. However, once the discussion involves the experimental relationships that were developed in the 1950s, current U.S. practice (English units) will be followed. The generalized equations presented in Paragraphs 5-3 and 5-4 will also use the U.S. convention, since that is the current practice here. Alternative forms of these equations in SI units are given in Appendix E. A second problematic area regarding unit conventions is how heat and radiation energy are treated. In the investigations described in *Snow Hydrology*, the heat quantity calorie was used, along with the radiation term langleys (calories/square centimeter). This convention has now been replaced by the use of joules, where 1 gram-calorie = 4.186 joules. Radiation flux is currently reported in several ways, as discussed in Appendix D. A conversion table is contained in Appendix C to assist in dealing with a somewhat confusing mixture of units.

c. Shortwave radiation melt. The applied equation component for shortwave radiation melt is taken directly from the theoretical equation for net radiation energy input at a point, Equation 2-4, combined with the general formula for snowmelt, Equation 2-2.

Thus

$$M_{sw(mm)} = \frac{1000(1-a)I_i}{334.9\ \rho B} \tag{5-1}$$

where

M_{sw} = daily shortwave radiation snowmelt, mm

a = snow albedo

I_i = daily incident solar radiation, kJ/(m^2 day)

ρ = density of water, 1000 kg/m^3

B = thermal quality of the snow

334.9 = latent heat of fusion of ice, kJ/kg

When the thermal quality is assumed to be 0.97 as discussed in Chapter 2, this equation reduces to

$$M_{sw(mm)} = 0.00308 I_i (1-a) \tag{5-2}$$

The alternative equation, when melt is expressed in inches and solar radiation is expressed in langleys, is obtained by employing the equivalent of Equation 5-1:

$$M_{sw(mm)} = \frac{(1-a)I_i}{(2.54)(80)B} \tag{5-3}$$

where

M_{sw} = daily shortwave radiation snowmelt, in.

I_i = daily incident solar radiation in langleys, cal/cm^2

2.54 = converts centimeters to inches

80 = latent heat of fusion of ice, cal/cm^3

This becomes

$$M_{sw} = 0.00508 I_i (1-a) \tag{5-4}$$

which becomes part of the generalized equation for melt (inches) in an open area presented in Paragraph 5-4.

d. Long-wave radiation melt. Long-wave radiation melt equations must consider, first, the radiation to the atmosphere from the snow surface, resulting in a net energy loss on clear days, and, second, the incoming (back) radiation emitted by the Earth's atmosphere, cloud cover, and forest canopy. Since the snow surface is nearly a perfect blackbody source of radiation, with a maximum temperature of 0 °C, long-wave radiation from the snow surface can be expressed as a constant employing the Stefan-Boltzmann equation. From Equation 2-5, using an emissivity of 0.99, this has been computed to be 0.315 kJ/(m^2 s). Using the older units of calories and langleys, the Stefan-Boltzmann coefficient is 8.26×10^{-10} ly/(min K^4), and the equation produces a long-wave radiation flux of 0.459 ly/min. This value is used in the generalized equation development that follows. It assumes an emissivitity of 1.0. Gray and Prowse (1992) note that emissivities can vary from 0.97 for dirty snow to 0.99 for clean snow.

(1) Back-radiation is a complex phenomenon involving factors such as the temperature of the cloud cover and tree canopy and the distribution of water vapor and temperature in the atmosphere. For that reason, experimental data and simplifying assumptions are used to develop relationships to express this. For back-radiation over snow under clear skies, the snow investigations experiments showed that a simple air temperature function can adequately express downward long-wave radiation because of the restricted range in vapor pressure normally experienced in these conditions. This equation is

$$Q_b = 0.76 \sigma T_a^4 \tag{5-5}$$

where

Q_b = long-wave radiation, ly/min

σ = Stefan-Boltzmann constant, ly/(min K^4)

T = air temperature, K

The net exchange by long-wave radiation is then:

$$Q_n = 0.76 \sigma T^4 - 0.459 \text{ (ly/min)} \tag{5-6}$$

(2) When clouds or forest cover are present, the back-radiation may be computed assuming that either is emitting radiation as a blackbody. Thus, the net long-wave radiation is computed by

$$Q_n = \sigma T^4 - 0.459 \quad \text{(ly/min)} \tag{5-7}$$

where T, the free air temperature, is assumed to approximate the temperature of the forest cover under surface or that of a low-elevation cloud base.

(3) The snowmelt resulting from long-wave radiation exchange is computed by combining Equations 5-6 and 5-7 with the general equation for melt, Equation 2-2. The resulting functions are nonlinear relationships between temperature (K) and long-wave radiation. In the snow investigation studies, these were simplified by fitting linear approximations and shifting to the Fahrenheit temperature scale. This is illustrated in Figure 5-1. The resulting equations for long-wave radiation melt are as follows.

(a) For melt under clear skies:

$$M_l = 0.0212 (T_a - 32) - 0.84 \tag{5-8}$$

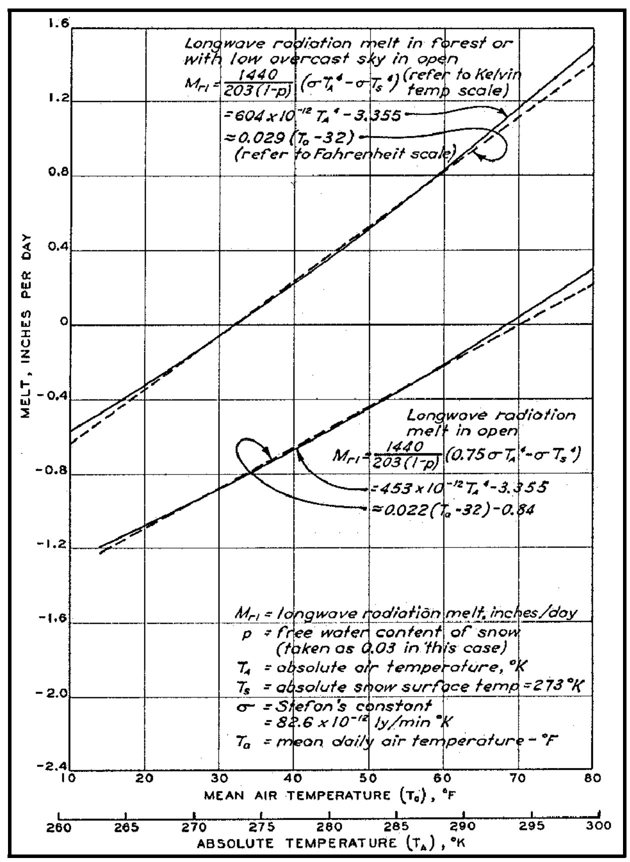

Figure 5-1. Linear adaptation of long-wave radiation functions (Figure 1, Plate 6-2, *Snow Hydrology)*

where M_l equals long-wave radiation melt (inches/day).

(b) For melt under a forest canopy or under a low cloud cover, as would be experienced during rain on snow,

$$M_l = 0.029(T_a - 32) \tag{5-9}$$

e. Convection (sensible heat) melt. Equation 2-6 is a general equation widely used to express the convective heat transfer between the air and snow surface. It represents a simplification of a complex physical process involving turbulent exchange taking place in the atmosphere 2 to 3 m above the snow surface. The key to this equation is the bulk transfer coefficient D_x, which has to be determined experimentally. As pointed out in Male and Gray (1981), there is a wide range of variation in the coefficient reported by researchers, so it is fortunate that the magnitude of this component of snowmelt is relatively small. This reference compares values from various sources including the snow investigations laboratories.

(1) The bulk exchange coefficient arrived at in the snow investigations program was based upon observations taken at the Central Sierra Snow Laboratory. Two other factors are also introduced to express the density of the atmosphere and to account for differences in the heights at which temperature and wind speed are measured. The resulting equation is similar in form to Equation 2-6 but is expressed directly in terms of snowmelt by applying the basic equation for snowmelt at a point (Equation 2-2):

$$M_c = 0.00629\left(\frac{p}{p_o}\right)(z_a z_b)^{-1/6}(T_a - T_s)v_b \tag{5-10}$$

where

M_c = convection melt, in./day

p, p_o = atmospheric pressures at location and at sea level, respectively

T_a = air temperature, °F

T_s = snow surface temperature, generally 0 °C (32 °F)

v_b = wind velocity, miles/hour

z_a,z_b = height of temperature and wind velocity measurement, ft

(2) For snow hydrology applications, Equation 5-10 was further simplified by assuming measurement heights of 3 and 15.2 m (10 and 50 ft) for air temperature and wind velocity, and by assuming a constant value of 0.8 for the atmospheric pressure ratio. This value would be considered appropriate for mountainous regions, with the range being 1.0 at sea level to 0.7 at a 3048-m (10,000-ft) elevation. With these assumptions, Equation 5-10 becomes

$$M_c = 0.00179 v_b (T_a - 32) \tag{5-11}$$

f. Condensation (latent heat) melt. The equation for computing condensation melt is similar in form to that for convection melt. Equation 2-7 defines the basic form, and the bulk transfer coefficient is determined from field measurements. The snow investigation studies led to the following equation based upon experimental analysis at the Central Sierra Snow Laboratory:

$$M_e = 0.054(z_a z_b)^{-1/6}(e_a - e_s)v_b \tag{5-12}$$

where

M_e = condensation melt, in./day

z_a,z_b = measurement heights, feet above snow surface for air vapor pressure and wind speed, respectively

e_a = vapor pressure of the air, in.

e_s = vapor pressure of the snow surface, mb

v_b = wind velocity, miles/hr

(1) Figure 5-2 is a plot of this equation, assuming a vapor pressure difference at 0.3 m (1 ft) above the

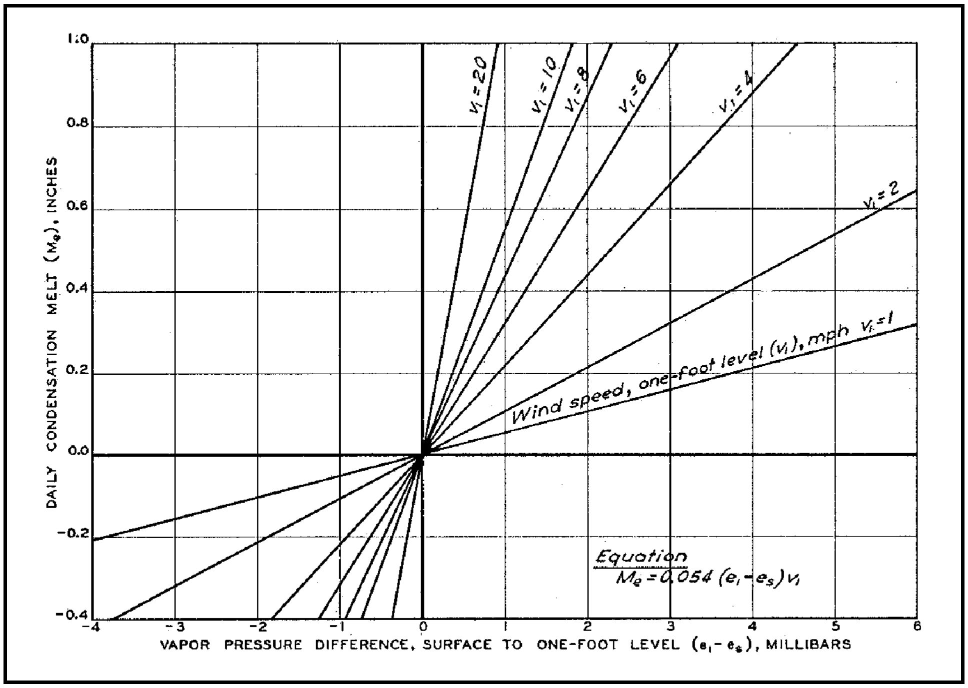

Figure 5-2. Daily condensation melt versus pressure gradient (Figure 2, Plate 5-5, *Snow Hydrology*

snow surface. Note the negative range of the function, indicating evaporation from the snow surface.

(2) Equation 5-12 can be simplified by assuming standardized measurement heights of 3 and 15.2 m (10 and 50 ft) above the snow surface, as was done with Equation 5-10. The other simplifying step is to replace vapor pressure with a variable that can be more practically measured and applied. A useful relationship exists between vapor pressure and dew-point temperature as shown in Figure 5-3. For the range of the variables normally encountered in practice, a linear approximation can be fitted:

$$e = 6.11 + 0.339(T_d - 32) \qquad (5\text{-}13)$$

where

e = vapor pressure, mb

T_d = dew-point temperature, ° F

6.11 = saturation vapor pressure, mb

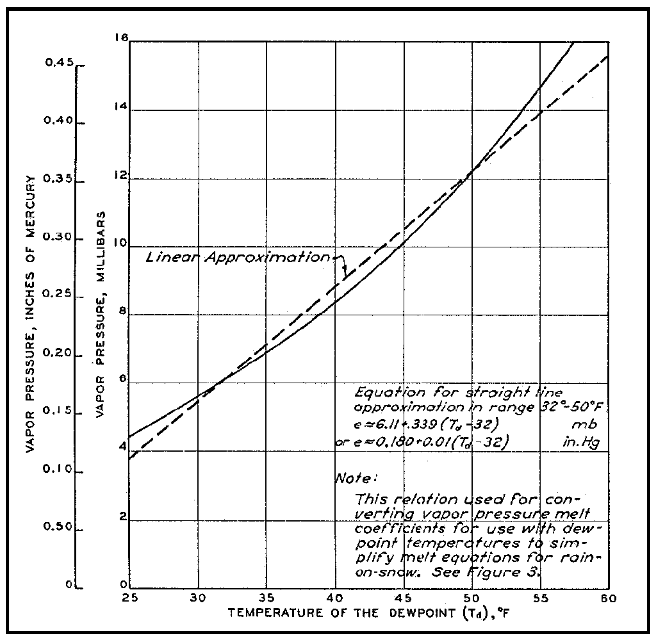

Figure 5-3. Experimental relationship between vapor pressure and dew point (Figure 5, Plate 6-2, *Snow Hydrology)*

(3) Combining Equations 5-12 and 5-13 and assuming the vapor pressure of the snow surface to be 6.11 mb, results in the simplified equation for condensation melt

$$M_e = 0.0065(T_d - 32)\, v_b \tag{5-14}$$

g. Combined convection-condensation equation. Since the equations for convection and condensation melt share some of the same variables, they are often shown in a combined form. Adding together Equations 5-11 and 5-14, the equation for combined daily snowmelt attributable to convection and condensation can be written:

$$M_{ce} = 0.0084v[0.22(T_a - 32) + 0.78(T_d - 32)] \tag{5-15}$$

where M_{ce} equals combined convection-condensation melt (inches per day).

In heavily forested areas where wind effects can be considered negligible, an alternative to Equation 5-15 for combined convection-condensation melt was determined experimentally:

$$M_{ce} = 0.045(T_a - 32) \tag{5-16}$$

h. Rain melt. Equation 2-9 is the basic formula expressing the heat energy given up when rainwater is cooled to the temperature of the snowpack, assuming the snowpack temperature is 0 °C. Using the following values for the coefficients,

$\rho = 1000\ \mathrm{kg/m^3}$

$C_p = 4.20\ \mathrm{kJ/(kg\ ^\circ C)}$

$T_s = 0\ ^\circ\mathrm{C}$

and applying Equation 2-2, this equation becomes

$$M_{r(mm)} = 0.0125 T_r P_r \tag{5-17}$$

where

M_r = daily snowmelt from heat supplied by rain, mm

T_r = temperature of rain, °C

P_r = daily rainfall, mm

The alternative to Equation 5-17 for English units is

$$M_r = 0.007(T_p - 32)\, P_r \tag{5-18}$$

where

M_r = daily snowmelt from heat supplied by rain, in.

T_r = temperature of rain, °F

P_r = daily rainfall, in.

i. Ground melt. The final source of energy for snowmelt is heat conducted from the ground. Once a snowpack becomes deep enough to insulate the ground from subfreezing air, an upward flux of heat can act to melt snow at the bottom of the snowpack. Although the rate of heat exchange is small, it can act continuously throughout a winter. As discussed in Chapter 2, a constant value is typically assumed for this component. Field experiments reported in *Snow Hydrology* and by Male and Gray (1981) estimate melt rates of 0.025 to 0.076 cm/day (0.01 to 0.03 in./day) ascribable to ground heat.

5-3. Generalized Equations, Rain-on-Snow Situations

a. Overview. For practical engineering use, the equations for snowmelt presented above can be combined into several generalized equations designated for specific meteorological and forest-cover conditions. Often the equations can be further simplified when the application is limited as specified. Also covered in this paragraph and in Paragraph 5-4 is the need to consider the equation being applied to a basin area rather than at a point, which is the basis for its derivation. This is accomplished by introducing constants representing the mean basin exposure to solar radiation or wind. This paragraph will present equations for use in rain-on-snow conditions, with varying degrees of forest cover. Paragraph 5-4 will introduce similar equations for rain-free applications.

b. Classification of forest density. The generalized equations presented below and in Paragraph 5-4 have been adopted to varying degrees of forest cover in the basin. Table 5-1 is a general guideline to follow in selecting the appropriate equation.

Table 5-1
Classification of Forest Density

Descriptive Category	Mean Canopy Cover, %
Heavily forested	>80
Forested	60-80
Partly forested	10-60
Open	<10

c. Basin wind exposure coefficient, k. For convection-condensation melt in basins, it is necessary to introduce a basin constant, *k*, that represents the mean exposure of the basin, or a segment of it, to wind, considering topographic and forest effects. For unforested plains, *k* would be 1, but for forested areas, the value may be as low as 0.3, depending upon the density of the forest stands. This factor can be estimated from topographic maps and aerial photographs but is best confirmed through model calibration.

d. Generalized equations. Snowmelt calculation in rain-on-snow settings is the simplest application of energy budget equations since solar radiation is minimal and the atmosphere can be assumed saturated, thereby simplifying the computation of convection and condensation melt. Two equations have been developed for rain-on-snow situations. The assumptions reflected in these equations follow and are summarized in Table 5-2 (Paragraph 5-5). Appendix E contains versions of these equations in SI units.

For open or partly forested basin areas,

$$M = (0.029 + 0.0084kv + 0.007P_r)(T_a\text{-}32) + 0.09 \qquad (5\text{-}19)$$

For heavily forested areas,

$$M = (0.074 + 0.007P_r)(T_a\text{-}32) + 0.05 \qquad (5\text{-}20)$$

where

M = snowmelt, in./day

k = basin wind coefficient

v = wind velocity, miles/hr

P_r = rate of precipitation, in./day

T_a = temperature of saturated air, at the 3-m (10-ft) level, °F

e. Open-partly forested basin equation. This equation is based upon simplified equations introduced in Paragraph 5-3. Shortwave radiation has been assumed constant at 0.127 cm/day (0.05 in./day), and ground melt is assumed to be 0.05 cm/day (0.02 in./day). Long-wave radiation uses Equation 5-9. The atmosphere is assumed to be saturated for these conditions, enabling the equating of dew-point temperature in Equation 5-15 to air temperature. This equation then becomes $M_{ce} = 0.0084\ (T_a - 32)$. Rain melt is computed with Equation 5-18, assuming that the rainwater temperature is equal to air temperature.

f. Heavily forested basin equation. Because of the dense forest cover, wind is assumed to be negligible in the convection-condensation equation. This permits using the alternative, Equation 5-16. A slight reduction is made in the assumed shortwave radiation to 0.076 cm/day (0.03 in/day).

g. Measurement height adjustment. As discussed in Paragraph 5-3, the convection and condensation equations reflect a simplifying assumption to the more

Table 5-2
Summary of Generalized Snowmelt Equations, Rain-on-Snow Situations

Equation	$M = (0.074+0.007P_r)(T_a-32)+0.05$	$M = (0.029+0.0084kv+0.007P_r)(T_a-32)+0.09$
Forest-Cover Application	Heavily forested (>80% cover)	Open to partly forested (10-80% cover)
Shortwave Radiation	• Very minor contribution • Assumed constant: 0.076 cm/day (0.03 in./day)	• Minor contribution • Assumed constant: 0.05 cm/day (0.02 in./day)
Long-wave Radiation	• Relatively important • Estimated as function of air temp.—factor is 0.029 in 0.074 coefficient • See Para. 5-2d; Equation 5-9 • Ref *Snow Hydrology* (SH), Ch. 6; Plate 6-2	• Relatively important • Estimated as function of air temp. (0.029) • See Para. 5-2d; Equation 5-9 • Ref. SH, Ch. 6, Plate 6-2
Convection-Condensation	• Relatively important melt component • Wind not a factor because of forest • Estimated as a function of air temp—factor is 0.045 in 0.074 coefficient • Conv. melt factor is $0.010T'_a$ • Cond. melt factor is $0.035T'_a$ • See Equation 5-16 • Ref SH, p. 231, Plate 6-2/Fig. 3	• Wind is an important factor • Estimated as a function of wind and air temp—coefficient = 0.0084 • Conv. melt factor = $0.0018T'_a v$ • Cond. melt factor = $0.0066T'_a v$ • Need to estimate k - basin exposure to wind. Varies 0.3 to 1.0 • Dew-point temp. assumed equal to air temp. (100% relative humidity) • See Equation 5-15 • Ref SH, Ch. 6, p. 231 • Ref Male and Gray (1981), pp. 385-393
Rain Melt	• Relatively small factor ($0.007P_rT'_a$) • Based upon heat content in rain, assuming rain temp. = air temp. • See Equation 5-18 • Ref SH, pp. 180, 230	• Relatively small factor ($0.007P_rT'_a$) • Based upon heat content in rain, assuming rain temp. = air temp • See Equation 5-18 • Ref SH, pp. 180, 230
Ground Melt	• Assumed constant: 0.05 cm/day (0.02 in./day)	• Assumed constant: 0.05 cm/day (0.02 in./day)

basic turbulent transfer equations that temperature and dew point and wind speed measurements are at 3 and 15.2 m (10 and 50 ft) above the snow surface, respectively. This assumption makes use of the relationship that defines the temperature and vapor pressure profiles as varying in height according to a 1/6 power function (*Snow Hydrology*, Chapter 5, USACE 1956). If measurements are made at heights other than the assumed 3 and 15.2 m (10 and 50 ft), the following adjustment factors can be used:

$$\text{Air temperature:} \quad CF_a = 1.47 Z_a^{1/6} \tag{5-21}$$

$$\text{Wind velocity:} \quad CF_b = 1.92 Z_b^{1/6} \tag{5-22}$$

where Z_a and Z_b are the height of the measurement above the snow surface in feet.

5-4. Generalized Equations, Rain-Free Situations

a. Overview. In rain-free settings, the calculation of snowmelt with energy budget equations must include solar radiation as a variable (unless there is heavy forest cover) in addition to the components considered in rain-on-snow situations. This introduces additional variables, such as albedo and cloud cover, as well as new factors that are needed to convert equations for melt at a point to a basin-mean relationship. Also, a saturated air mass can no longer be assumed, thus requiring use of dew point as a variable. These variables and coefficients will be described in this chapter, and the generalized equations will be presented along with a summary of the assumptions reflected in each equation. A tabular summary (Table 5-3) is presented.

Table 5-3
Summary of Generalized Snowmelt Equations, Rain-Free Situations

Equation	$M=0.074(0.53T'_a+0.47T'_d)$	$M=k(0.0084v)(0.22T'_a+0.78T'_d)+0.029T'_a$
Forest Cover Application	Heavily forested (>80% cover)	Forested (60-80% cover)
Shortwave Radiation Melt; Ground Melt	• Relatively unimportant; assumed compensated for by evapotranspiration	• Relatively unimportant; assumed compensated for by evapotranspiration
Long-Wave Radiation Melt	• Relatively important • Estimated as function of air temp.—factor is $0.029T'_a$ • See Para. 5-2d, Equation 5-9 • Ref SH, Plate 6-2	• Relatively important • Estimated as function of air temp.—factor is $0.029T'_a$ • See Para. 5-2d, Equation 5-9 • Ref SH, Plate 6-2
Convection-Condensation Melt	• Relatively important • Wind not a factor because of forest cover • Conv. estimated as a function of air temp.—factor is $0.011T'_a$ • Cond. estimated as a function of dew-point temp.—factor is $0.035T'_d$ • Ref SH, Plate 6-2/Fig. 3	• Relatively important • Wind is an important factor • Conv. estimated as a function of air temp. and wind—factor is $0.0018T'_a v$ • Cond. estimated as a function of dew-point temp. and wind—factor is $0.0066T'_d v$ • Need to estimate k - basin exposure to wind. Varies 0.3 to 1.0 • See Para. 5-2e,f; Equations 5-11, 5-13 • Ref SH, Plate 6-2/Fig. 3 • Ref Male and Gray (1981), pp. 385-393
Equation	$M=k'(1\text{-F})(0.0040I_i)(1\text{-}a)$ $+k(0.0084v)(0.22T'_a+0.78T'_d)$ $+F(0.029T'_a)$	$M=k'(0.00508I_a)(1\text{-}a)$ $+(1\text{-}N)(0.0212T'_a\text{-}0.84)$ $+N(0.029)T'_c$ $+k(0.0084v)(0.22T'_a+0.78T'_d)$
Forest Cover Application	Partly forested (10-60%)	Open (<10%)
Shortwave Radiation Melt	• Important factor • Function of solar insolation and albedo for unforested portions of the basin • Need estimate of k' factor (see Para. 5-4d) • Long-wave loss for open areas reflected in the shortwave coefficient, 0.004 • See Para. 5-4c re: albedo • See Para. 5-4e re: forest-cover factor, F • See Para. 5-4h • Ref SH, pp. 212-214	• Important factor • Function of solar insolation and albedo • Uses theoretical melt equation (see Equation 5-4) • Need estimate of k' factor (see Para. 5-4d) • See Para. 5-2c • Ref SH, pp. 212
Long-Wave Radiation Melt	• Relatively important factor • For forested area: function of air temp.—factor is $0.029T'_a$ • For unforested area: computed indirectly by reducing SW melt factor • See Para. 5-4e re: forest-cover factor, F • See Para. 5-2d, Equation 5-9, Para. 5-4h • Ref SH, Plate 6-2	• Important factor—loss in clear areas • Computed directly for cloud-free areas—factor is $(0.0212T'_a\text{-}0.84)$ • See Para. 5-2d, Equation 5-8 • Ref SH, Plate 6-2/Fig. 1
Convection-Condensation Melt	• Less important compared with SW melt • Computed as in forested area equation	• Less important compared with SW melt • Computed as in forested area equation

b. Solar radiation. This variable, discussed in Paragraphs 2-2 and 5-3 and Appendix D, needs to be specified as input unless there is heavy forest cover. The following two basic approaches are used in preparing solar-radiation input.

(1) Observations of solar radiation are made at first-order National Weather Service stations in the United States. These data are available from regional and national NWS archives. The data are reported as insolation (shortwave solar radiation on a horizontal surface). Since there are relatively few stations making these observations, it is unlikely that historical observations would be used directly as model input (for model calibration, for instance); however, such data could be used to estimate a historical time series or to help construct a hypothetical time series for a design flood derivation.

(2) Equations, charts, and nomographs have been developed that can be used to construct hypothetical time series of daily solar radiation or as the basis for estimating maximum theoretical insolation for historical conditions. These generally involve a theoretical insolation quantity that is based upon latitude and time of year, then corrected for transmittivity through the atmosphere. Reference is made to Appendix D, which contains a chart that could be used for this, and to Male and Gray (1981). It is necessary to establish a reasonable relative magnitude for solar radiation that is consistent with the engineering application involved, e.g., a maximized sequence in the case of a probable maximum flood (PMF) derivation. The key variable affecting the quantity of solar radiation is cloud cover, once the location and time of year are established. The appropriate amount of cloud cover could be estimated by referring to historical records of sunshine duration, diurnal temperature, cloud cover, etc. Figure 5-4 is a plot, derived from Figure D-8, showing the effect of cloud cover on insolation, given a known theoretical solar radiation amount based upon latitude and time of year. An example of solar radiation sequence developed for a PMF derivation is described in Chapter 10.

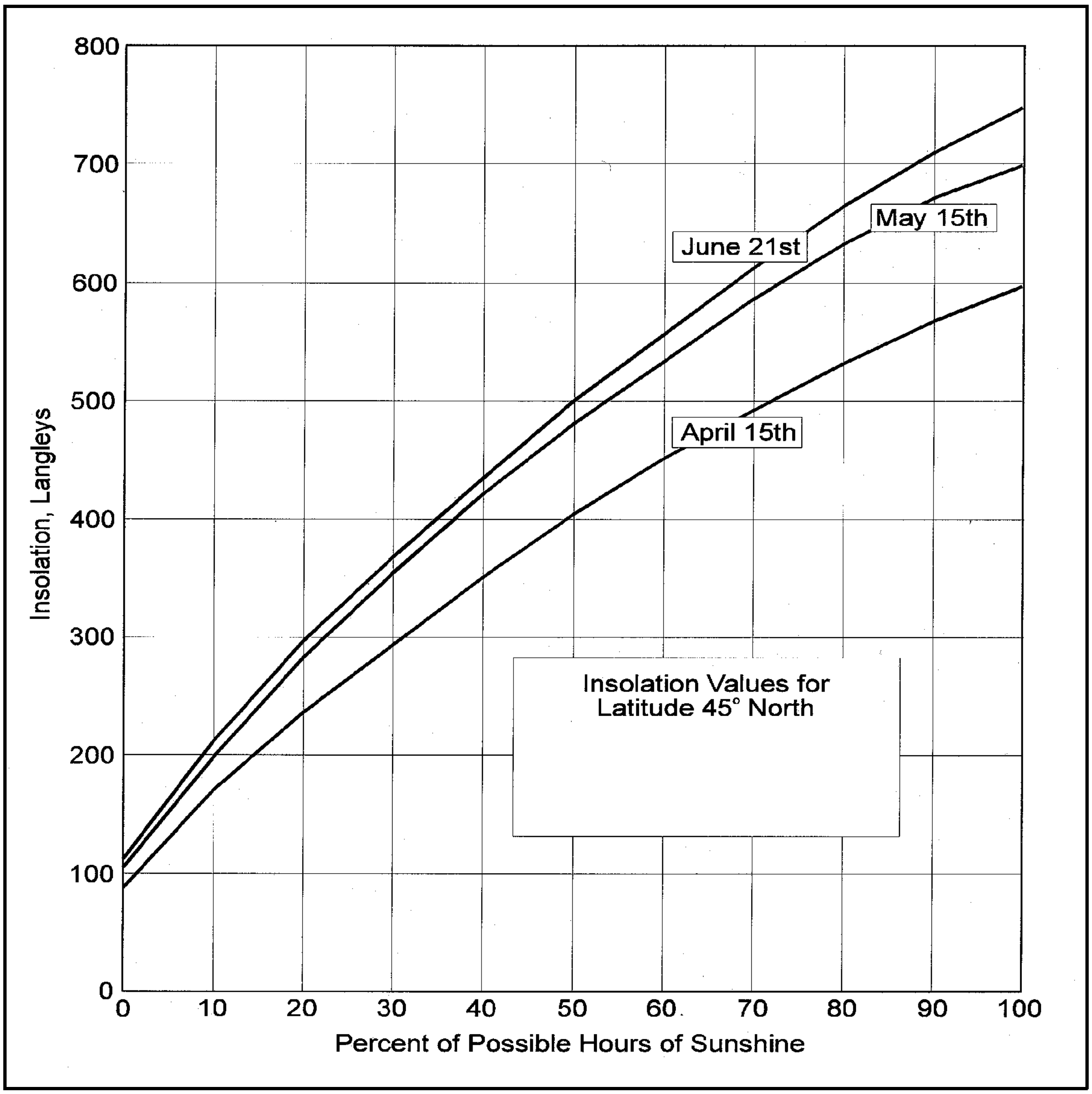

Figure 5-4. Effect of cloud cover on solar (shortwave) radiation

c. Snow surface albedo. Since there are no regular observations of snow surface albedo, this variable must be estimated on the basis of relationships established in laboratory experiments. Figure 5-5 shows a typical variation of snow surface albedo with time, for both the accumulation and melt seasons. This illustrates the general phenomenon involved, that albedo decreases as the snowpack ripens. In computer simulations this can be expressed as a decay function in the form (Laramie and Schaake 1972):

$$a = e(f)^{N^g} \tag{5-23}$$

where

a = snow surface albedo

N = number of elapsed days

e, f, g = experimental coefficients

d. Basin shortwave melt coefficient, k′. Measurements of solar radiation are generally expressed in terms of amounts on a horizontal surface. For basins whose exposure is predominately north- or south-facing, a basin shortwave melt coefficient must be introduced in the melt equation. Reference is made to Figure D-6, showing the effect on incident solar radiation of a 25° slope at latitude 46°30′ N. In general, averaged over a basin, the slope effect would not be as extreme as the particular example shown in this figure. The value of k' would be 1.0 for a basin that is essentially horizontal or whose north and south slopes are areally balanced. The value of k' usually would fall within the limits of 0.9 and 1.1 during the spring.

e. Effective forest canopy cover, F. For partly forested basins, it is necessary to estimate the effective forest canopy cover F, which is applied to determine shortwave and long-wave radiation snowmelt. The coefficient F represents the average proportion of the basin shaded by the forest from direct solar radiation, expressed as a decimal fraction. Determination of F must be based upon a partly subjective estimate of the forest characteristics, considering density and spacing of forest stands, latitudinal, and diurnal effects of the forest upon shading, and general knowledge gained from personal observation or remote sensing

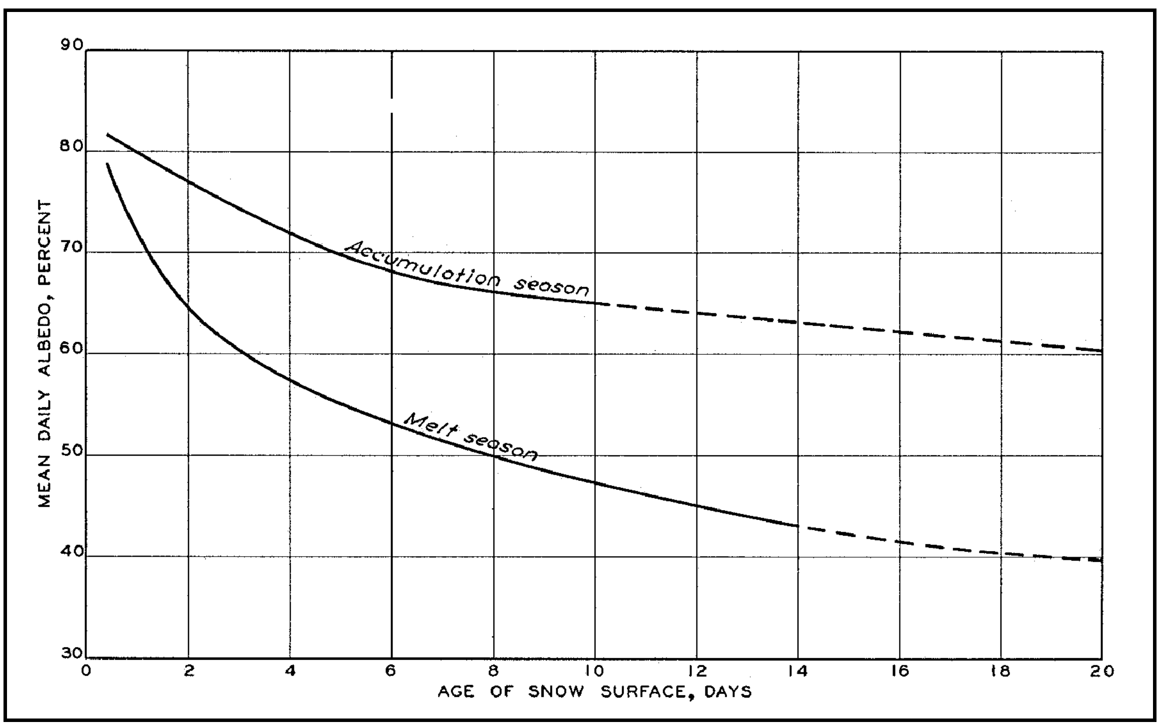

Figure 5-5. Albedo versus age of snow (Figure 4, Plate 5-2, *Snow Hydrology*)

photography. In general, the value of F is somewhat greater than the theoretical cover taken as the horizontal projection of the forest canopy (see Figure D-7).

f. Generalized equations. Four equations have been developed for the four categories of forest cover presented in Table 5-1.

(1) For open areas

$$M = k' (0.00508 I_i)(1-a) + (1-N)(0.0212\, T'_a - 0.84) + N(0.029 T'_c) + k(0.0084 v)(0.22\, T'_a + 0.78 T'_d) \quad (5\text{-}24)$$

(2) For partly forested areas

$$M = k'(1-F)(0.0040 I_i)(1-a) + k(0.0084)(0.22 T'_a + 0.78 T'_d) + F(0.029 T'_a) \quad (5\text{-}25)$$

(3) For forested areas

$$M = k'(0.0084 v)(0.22 T'_a + 0.78 T'_d) + F(0.029 T'_a) \quad (5\text{-}26)$$

(4) For heavily forested areas

$$M = 0.074(0.53 T'_a + 0.47 T'_d) \quad (5\text{-}27)$$

where

M = snowmelt rate, in./day

T'_a = difference between the air temperature measured at 3 m (10 ft) and the snow surface temperature, °F

T'_d = difference between the dew-point temperature measured at 3 m (10 ft) and the snow surface temperature, °F

v = wind speed at 15.2 m (50 ft) above the snow surface, miles/hr

I_i = insolation (solar radiation on a horizontal surface, langleys

a = average snow surface albedo, decimal fraction

k' = basin shortwave radiation melt factor

F = average basin forest-canopy cover shading of the area from solar radiation, decimal fraction

T'_c = difference between the cloud base temperature and snow surface temperature, °F

N = estimated cloud cover expressed, decimal fraction

k = basin convection-condensation melt factor expressing average exposure to wind

g. Open-area equation. This equation is based upon theoretical principles, with coefficients determined on the basis of observations at a lysimeter at the Central Sierra Snow Laboratory. Shortwave radiation, usually always the most important melt factor in this setting, is based upon the measured or assumed incident solar radiation (taking into account cloud-cover estimates), together with snow surface albedo and the basin shortwave radiation melt coefficient k'. Long-wave radiation is calculated on the basis of the air temperature relationship (Equation 5-8) for cloud-free periods. For periods with cloud cover, Equation 5-9 is applied, using the difference between the temperature of the cloud base and the snow surface temperature. The cloud base temperature can be estimated from upper air temperatures or from lapse rates from a surface station. Convection and condensation, usually of relatively minor importance in this setting, are computed using Equation 5-15.

h. Partly forested area equation. This equation and those following reflect a different method of derivation from the procedures used for Equation 5-24 and for the rain-on-snow equations. Instead of relying entirely on a theoretical factors, multiple regression techniques employing field-laboratory data were used to establish some of the coefficients for a given forest cover. Thus, in the treatment of shortwave radiation and long-wave loss, the long-wave loss is computed indirectly by incorporating it into the statistically derived shortwave radiation coefficient. This results in a coefficient of 0.0040 compared with the theoretical value of 0.00508. The shortwave radiation is computed only for nonforested areas, using the effective forest canopy factor *F*.

i. Forested-area equation. Equation 5-26 reflects the assumption that shortwave radiation is unimportant because of the forest cover. The basin-mean wind, however, is assumed to be significant enough to effect convection-condensation melt and is computed as in Equation 5-25. Long-wave radiation from the forest canopy is computed as a function of air temperature as in Equation 5-25.

j. Heavily forested-area equation. This equation is obtained from correlation analysis of data for the Willamette Basin Snow Laboratory, a heavily forested field site. (See Table 5-3 for specific references.) The convection melt term is $0.011(T_a - 32)$; the long-wave radiation term is $0.029(T_a - 32)$; and the condensation melt term is $0.034(T_d - 32)$. Combining these terms yields Equation 5-27.

5-5. Summary of Generalized Energy Budget Equations

Tables 5-2 and 5-3 summarize the energy budget equations for rain-on-snow and rain-free situations.

5-6. Sensitivity of Variables and Coefficients in Generalized Equations

a. Overview. This section discusses the relative magnitude of the snowmelt components described in Paragraph 5-3 and contained within the generalized equations presented in Paragraph 5-4, including further analysis of the sensitivity of the variables and factors inherent in the equations. The discussion is intended to assist the practitioner in applying either the energy budget equations or temperature index procedures (Chapter 6) for snowmelt analysis and simulation. The paragraph addresses questions such as which factors are most important in given meteorological and geographical settings and where emphasis should be placed in obtaining data and performing the analysis.

b. Relative magnitude of melt components. The energy budget equations provide a convenient means for illustrating the relative magnitude of the snowmelt components for specified conditions. In Table 5-4, seven assumed settings are postulated, together with the assumed meteorological conditions. Three are for rain-on-snow and three are for rain-free conditions. The melt quantities are computed using the appropriate generalized equation. For the rain-free condition, the melt quantities illustrate the importance of shortwave radiation as a melt-producing source and also show how cloud cover and albedo changes can significantly affect this melt component. For the rain-on-snow condition, the dominance of condensation melt can be seen, along with the importance that wind velocity plays in this component. Rain melt, by contrast, is relatively small, even for the condition having relatively heavy rainfall.

(1) Cases 1 and 3 illustrate the effects of cloud cover in a rain-free situation. Two factors are at work: first, shortwave radiation is reduced because of cloud cover, and second, net long-wave radiation is increased as outgoing radiation is decreased and back-radiation from clouds is increased. These two melt components therefore tend to offset themselves. This suggests that cloud cover is a somewhat insensitive variable in the overall equation once the maximum possible insolation rate is established for the time of year and latitude.

(2) Further illustration of the relative contribution from the energy budget components is shown in Figures 5-6 through 5-8. For these relationships, daily snowmelt has been computed from the appropriate generalized equation and plotted against air temperature as the main independent variable (*x*), with a second variable as a parameter (*z*). They illustrate the variability and importance of the second variable compared with the most frequently used index variable, air temperature. These plots will be referred to again in the discussion of the temperature index method (Chapter 6). Figure 5-6 shows that, during rain-on-snow, precipitation magnitude does not introduce a significant additional variance in melt over that supplied by air temperature. In Figure 5-7, wind velocity—affecting convection and condensation—is an important variable in computing snowmelt, having almost the amount of variance as temperature. Figure 5-8 shows the effect of wind velocity in a partly forested rain-free setting. Note the lower magnitudes of melt in comparison with Figure 5-7, because condensation melt is

Table 5-4
Magnitude of Melt for Identifiable Meteorological Settings

a. Assumed Conditions

Case	Description	Assumed Meteorological Conditions T_a	T_d	I_i	P_r	v
1.	Clear, hot, summer day. No forest cover. Albedo = 40%	70	45	700	0	3
2.	Same as Case 1, 40% forest cover	70	45	700	0	3
3.	Same as Case 1, 50% cloud cover	65	50	500	0	3
4.	Same as Case 1, fresh snow. Albedo = 70%	70	45	700	0	3
- -	- -	- -	- -	- -	- -	- -
5.	Heavy wind and rain, warm. No forest cover	50	50	0	3	15
6.	Same as Case 5, but light rain, windy	50	50	0	0.5	15
7.	Same as Case 6, but light wind	50	50	0	0.5	3

B. Daily Melt Quantities

Case	Snowmelt components, in. M_{sw}	M_l	M_{ce}	M_r	M_g	Total Melt in.	Rain + Melt in.
1.	2.13	-0.03	0.47	0	0	2.57	2.57
2.	1.01	0.44	0.28	0	0	1.73	1.73
3.	1.52	0.34	0.54	0	0	2.40	2.40
4.	1.07	-0.03	0.47	0	0	1.51	1.51
- -	- - -	- - -	- - -	- - -	- - -	- - -	- - -
5.	0.05	0.52	2.27	0.38	0.02	3.24	6.24
6.	0.05	0.52	2.27	0.06	0.02	2.92	3.42
7.	0.05	0.52	2.27	0.06	0.02	1.11	1.61

Note: T_a = Air temperature, °F; T_d = Dew-point temperature, °F; I_i = Solar insulation, langleys; P_r = Daily rainfall, in.; v = Mean wind velocity, mph; M_{sw} = Shortwave radiation melt; M_l = Long-wave radiation melt; M_{ce} = Convection/condensation melt; M_r = Rain melt; M_g = Ground heat melt.

not as significant a factor in these dry conditions, and convection is a relatively small component.

c. Sensitivity of coefficients. In the previous discussions introducing the energy budget equations, the basis for the various factors and coefficients in the equations have been explained. Some are based upon theoretical principles; others are strictly empirical and perhaps vary a great deal. The degree of influence on the final outcome of the equation largely depends, of course, on the importance of the variable in the equation with which a coefficient or constant is associated. A summary of the most important and critical factors to be concerned about is listed below and are also noted in Tables 5-2 and 5-3.

(1) In the equations that take into account wind velocity in computing convection-condensation melt, the factors associated with the term become quite sensitive in influencing the computed melt. In a partly forested rain-on-snow situation, for instance, the convection-condensation term carries the most weight in determining melt, over 50 percent when wind velocity is relatively great. This places considerable importance on the wind exposure constant *k*, which can have a wide range of variation. As previously noted, that part of the coefficient 0.0084 pertaining to convection melt is also subject to wide variation as an experimental coefficient. These factors, therefore, can be considered to be sensitive and should be treated with care if they are subjectively determined. This concern can be reduced when using a model that can be calibrated and verified with historical data.

(2) In the rain-free equations, the dew-point variable is often not directly available and might be estimated on the basis of assumptions of relative humidity magnitude

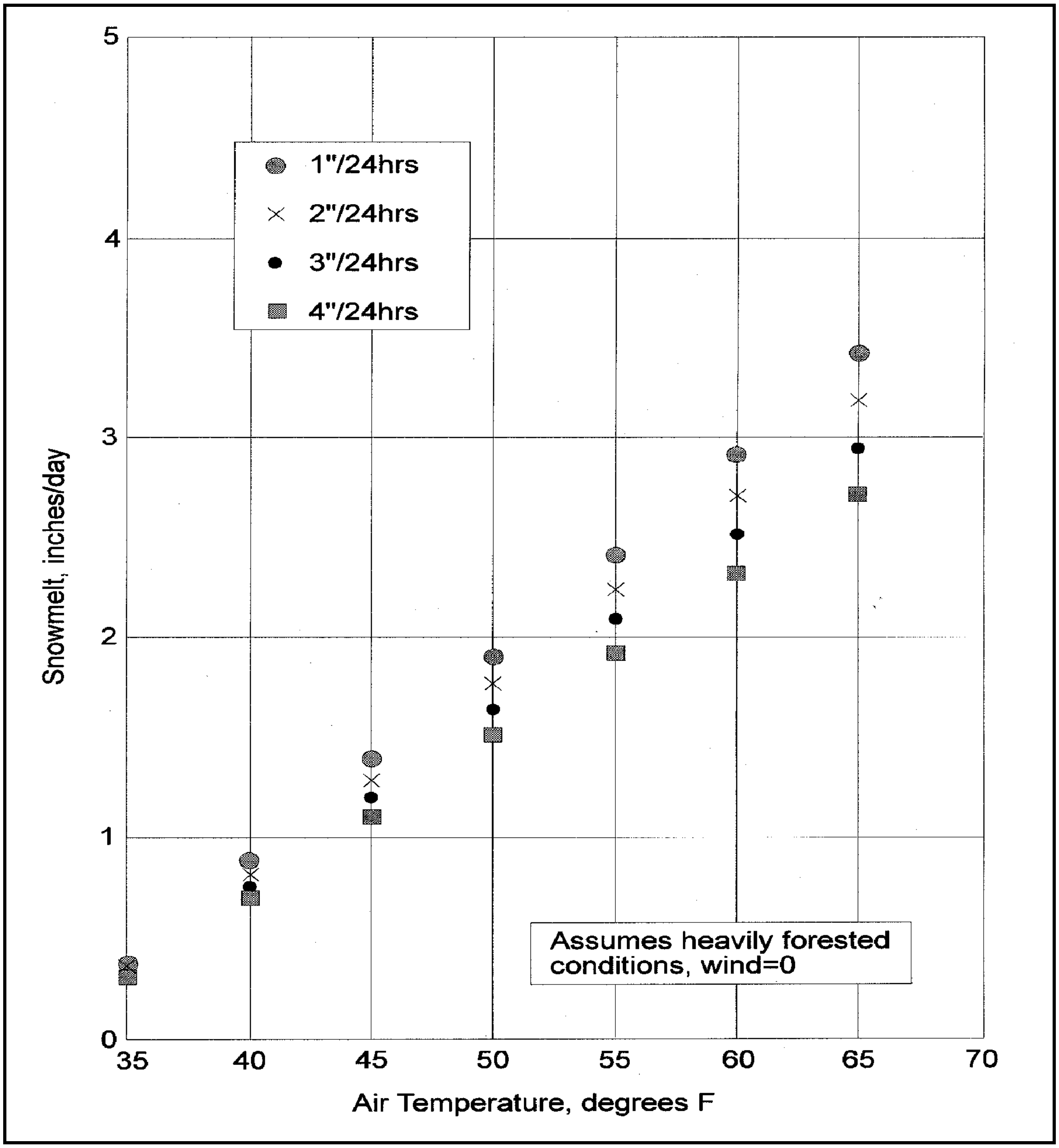

Figure 5-6. Melt versus temperature and precipitation, rain-on-snow area

and air temperature (Appendix D). Since condensation melt, which this variable indexes, can be one of the more influential components in computing melt, the dew point must be carefully estimated.

(3) In the equations that use solar radiation as a variable, the solar radiation term often becomes the most significant term in the melt equation. Thus, the factors k' and F become relatively important, as does albedo. The factor k' (shortwave radiation melt factor) is defined as being relatively insensitive, varying between 0.9 and 1.1. The forest-canopy cover factor F is a measurable factor that is therefore limited in its variability. The snow surface albedo, which must be calculated or estimated, can be quite variable in real-time during periods of snow accumulation, but should follow a relatively predictable decay function once snow ablation is underway. This factor is quite significant in affecting solar radiation melt magnitude as

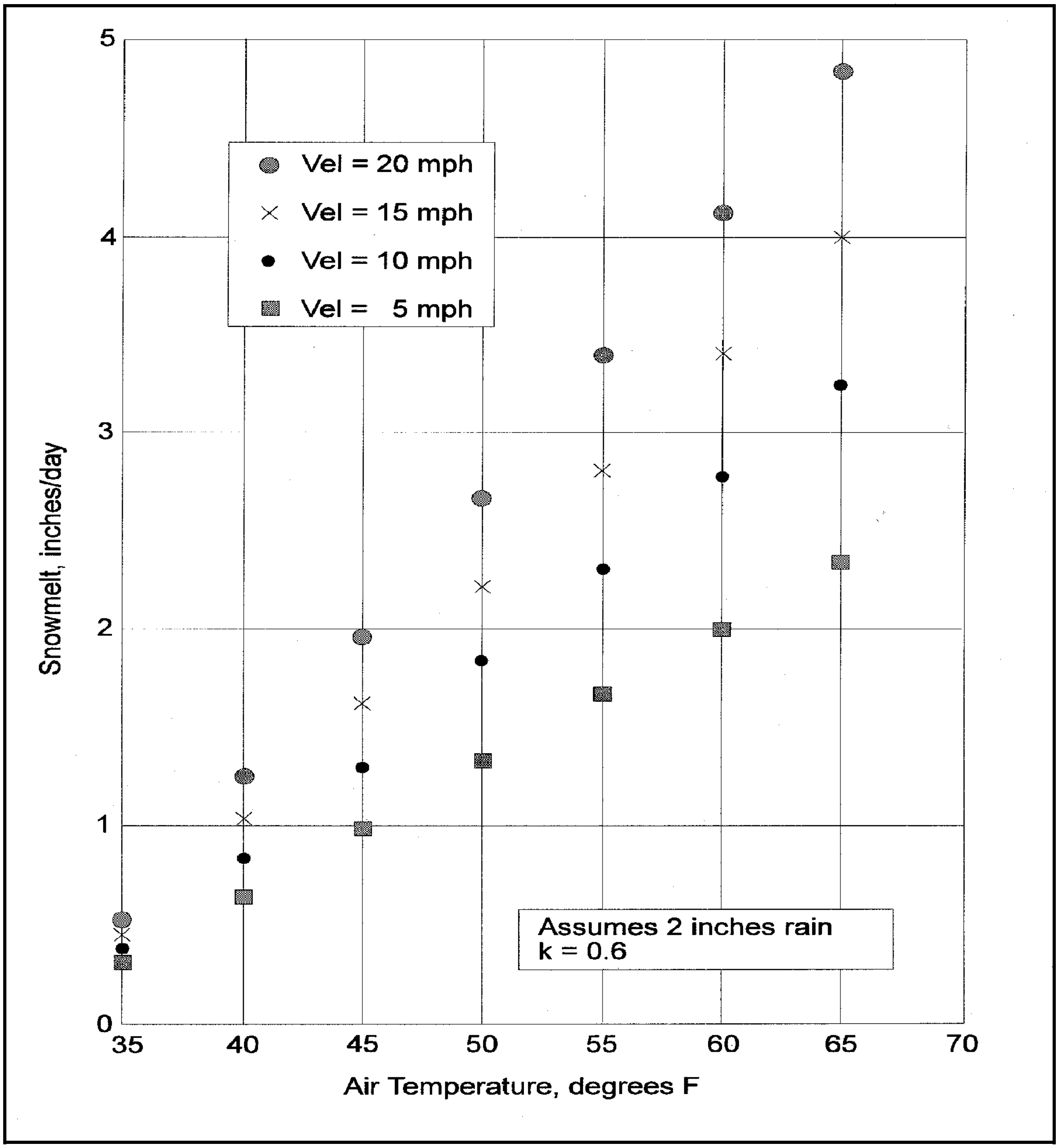

Figure 5-7. Melt versus wind and temperature, rain-on-snow area

demonstrated in Table 5-4. The coefficient 0.0040 in the partly forested equation has been determined by statistical means and, as discussed in *Snow Hydrology*, appears to have shown relatively good consistency when computed from different laboratory data.

(4) In general, care must be taken in choosing one equation over another on the basis of forest cover. A borderline forest-cover percentage could lead to quite different melt quantities, depending upon which equation was applied.

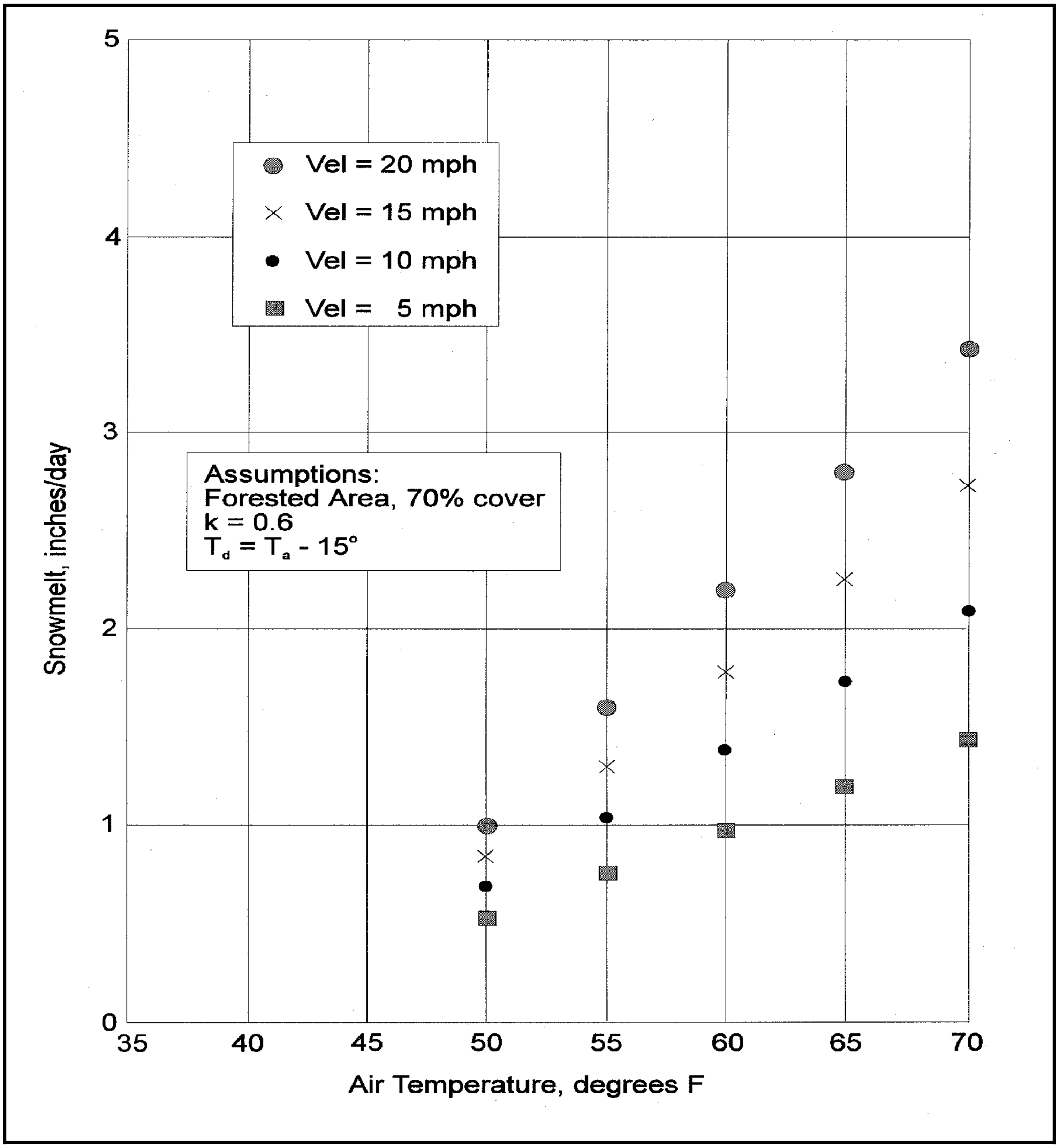

Figure 5-8. Melt versus wind and temperature, rain-free, partly forested area