APPENDIX-C
GENERATION OF ISOTHERMS

C-1. General. Generation of an isotherm from laboratory data is straightforward. The isotherm is plotted in log-log form to ensure that it approaches a straight line, making it easier to read and interpret, and to apply the Freundlich equation. After the data are plotted on a log-log scale, we can define the equation of the line by taking the log of each side of the Freundlich equation. This allows you to determine the values of K and $1/n$. The logarithmic version of the Freundlich equation is a straight line in the form of $y = mx + b$, where b is the y-intercept for $x = 0$. In our case K equals the x/m intercept when the log of the concentration C (adsorbate remaining) equals one unit, the log of which is zero. The value of m, the slope of the line, is equal to $1/n$, which can be solved by rearranging the equation to:

$$\frac{1}{n} = (\log \frac{x}{m} - \log K) \ / \ (\log C)$$

There are two ways you can generate the isotherms:

- From laboratory data.
- From existing data (see Figure 2-2).

a. Isotherm Generation from Existing Data. Given TCE as the contaminant,

$K = 1060$ µg/g, $\frac{1}{n} = 0.500$.

To generate an isotherm from

$$\log \frac{x}{m} = \log K + \frac{1}{n} \log C$$

we can rewrite to

$$\log \frac{x}{m} = \log 1060 + 0.500 \log C$$

$$\log \frac{x}{m} = 3.025 + 0.500 \log C$$

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and for various values of *C,* plot an isotherm. Try

C = 1, l0, l00, 1000 µg/L

$\log \frac{x}{m}$ = 3.025, 3.525, 4.025, 4.525

$\frac{x}{m}$ = 1060, 3350, 10600, 33500

Which results in the curve shown in Figure C-1.

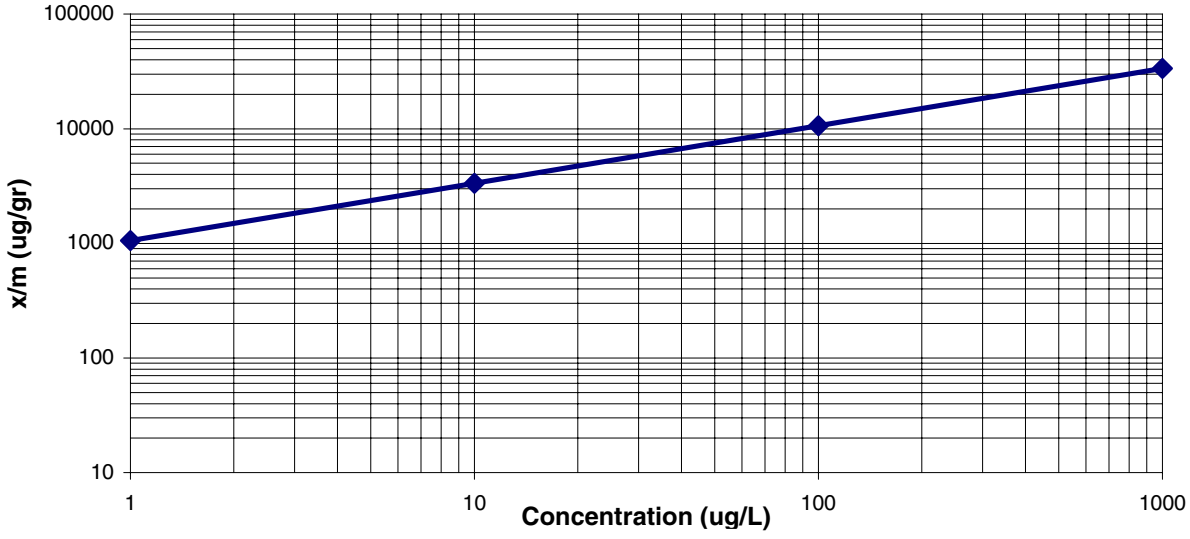

Figure C-1. TCE isotherm.

b. Example Isotherm Problem. The following laboratory data were collected in a batch adsorption study. Plot the data according to the Freundlich isotherm and determine the values for the constants 1/*n* and *K*. A volume of 500 mL is placed in each flask, and the waste has an initial concentration of 100 µg/L.

Flask Number	Carbon Weight (mg) ***(m)***	Flask Volume (ml)	Final TOC (mg/L) **(C)**	Adsorbate Adsorbed (mg) ***(x)***	x/m ***(mg/g)***
1	965	500	3.5	48.25	50.00
2	740	500	5.2	47.40	64.05
3	548	500	8.0	46.00	83.94
4	398	500	12.5	43.75	109.92
5	265	500	20.5	39.75	150.00
6	168	500	33.0	33.50	199.40
7	0	500	100.0	0	0

Example Calcs:

x= (100 mg/L - 3.5 mg/L)*(500 mL volume/1000 mL/L) = 48.25 mg

x/m= (48.25 mg/0.965 g) = 0.05mg adsorbate/mg of Carbon

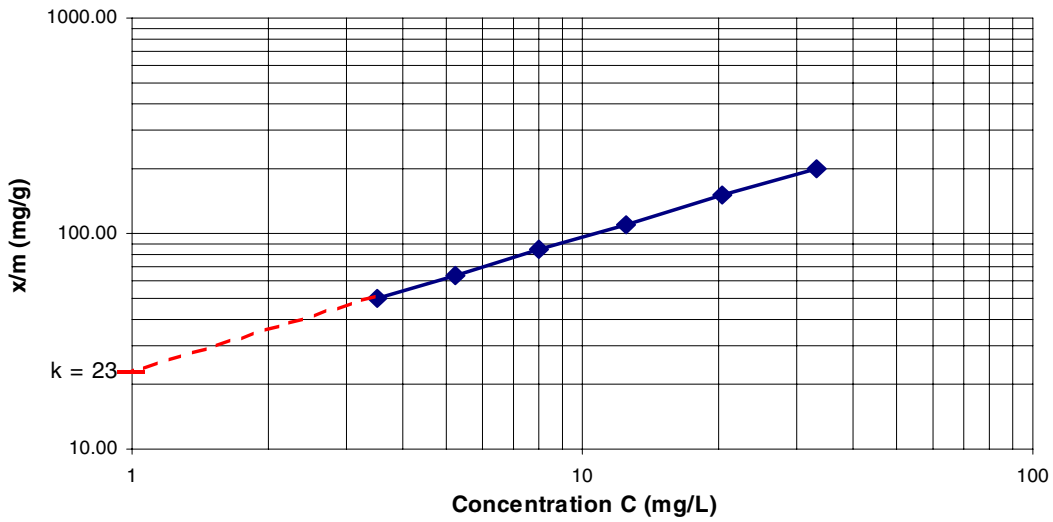

Figure C-2. TOC isotherm and lab data.

c. Solution.

(1) Calculate the values of *x* and *x/m* from the data. Refer to Figure C-1.

(2) Plot values of *x/m* vs. *C* on log-log paper in a manner similar to that shown in Figure C-1 (see Figure C-2).

(3) Determine the values of the constants K and $1/n$.

(a) To determine the intercept of a line on a log-log plot, the value of the intercept must read at the point where the value of the abscissa is equal to 1.0. To determine K from Figure C-2, locate a value of $C = 1.0$ and read $K = 23.0$.

(b) The slope of the line will yield a value of $1/n$. The slope of a log-log plot can be determined by scaling or by the following calculations.

$$\log \frac{x}{m} = \log K - \frac{1}{n} \log C$$

$$\frac{1}{n} = \frac{\log \frac{x}{m} - \log K}{\log C}$$

For a point on the line at $x/m = 0.70$, $C = 6$

$$\frac{1}{n} = \frac{\log 0.07 - \log 0.023}{\log 6} = \frac{-1.154 - (-1.638)}{0.778}$$

$$\frac{1}{n} = 0.622$$

$$n = 1.607$$

(c) The Freundlich equation then becomes

$$\log \frac{x}{m} = \log 0.023 - \frac{1}{1.607} \log C$$

$$\log \frac{x}{m} = 1.638 - 0.622 \log C$$